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AUTHORITATIVE DISCOURSE IN THE MIDDLE SCHOOL MATHEMATICS CLASSROOM: A CASE STUDY

A Dissertation

by

ADAM P. HARBAUGH

Submitted to the Office of Graduate Studies of
Texas A&M University
in partial fulfillment of the requirements for the degree of

DOCTOR OF PHILOSOPHY

August 2005

Major Subject: Curriculum and Instruction

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Approved by:

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ABSTRACT

Authoritative Discourse in the Middle School Mathematics Classroom: A Case Study.

(August 2005)

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According to the National Council of Teachers of Mathematics (NCTM) standard of communication, “Instructional programs from pre-kindergarten through grade 12 should enable all students to...communicate their mathematical thinking coherently and clearly to peers, teachers, and others” and students need to learn “what is acceptable as evidence in mathematics” (NCTM, 2000, p. 60). But do teachers have a clear understanding of what is acceptable or do they believe that the only acceptable explanations are the ones that they themselves gave to the students? Can teachers accept alternative forms of explanation and methods of solution as mathematically accurate or do they want students to simply restate the teachers’ understandings of mathematics and the problem? The focus of this dissertation is the authoritative discourse practices of classroom teachers as they relate to individual students and large and small groups of students.

In this case study, I examine the interactions in one eighth-grade mathematics classroom and the possible sharing of mathematical authority and development of mathematical agency that take place via the teacher’s uses of authoritative discourse. A guiding objective of this research was to examine the ways a teacher's discursive practices were aligned with her pedagogical intentions.

The teacher for this study was an experienced eighth-grade mathematics teacher at a rural Central Texas middle school. The teacher was a participant in the Middle School Mathematics Project at Texas A&M University. Results of an analysis of the discourse of six selected classes were combined with interview and observation data and curriculum materials to inform the research questions.

I found that through the teacher's regular use of authoritative discursive devices, mathematical authority was infrequently shared. Also the teacher's uses of authoritative discourse helped create an environment where mathematical agency was not encouraged or supported. The teacher's use of various discursive devices helped establish and maintain a hierarchy of mathematical authority with students at the lowest level reliant on others for various mathematical decisions.

DEDICATION

To my father, who would have enjoyed this, and to Rebecca and Sarah, with love

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I would like to first thank my wife, Rebecca, without whom I could not make it through life, let alone a dissertation. Your loving support and encouragement throughout this process has made it a much easier road to travel. Thank you for continuing to remind me of what is important. Thank you to my daughter, Sarah, who makes me laugh everyday. I am amazed and inspired by everything about you. I hope that one day you like mathematics as much as I do.

To the members of my committee, thank you all for each of the many contributions you have made to my education and this dissertation. Thank you, Dr. Gerald Kulm. It took a little while for me to get to know you, but I am certainly grateful that I did. You have opened many doors for me through your wisdom, guidance, and support. Thank you, Dr. Mary Margaret Capraro. You have been a friend and teacher from the beginning. Your support has many faces and each of them is appreciated. Your commitment to mathematics education was and continues to be an inspiration to me and all you touch through your work ethic and care. To Dr. Robert Capraro, what can I say? Your insight and guidance have provided me with such an important part of my education as a student and scholar. Your friendship has been invaluable. I would not have even pursued this had it not been for your suggestions. To Dr. Bruce Lowe, thank you for being there when I needed you most. Your laughter is literally contagious.

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Finally, thank you to all the teachers of whom I was student, from grade-school to college. You have, in various ways, helped to shape the course of my life. I will always be grateful for teachers.

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INTRODUCTION

> A careful analysis of the teacher-student relationship at any level, inside or outside the school, reveals its fundamentally narrative character. This relationship involves a narrating Subject (the teacher) and patient, listening objects (the students). The contents, whether values or empirical dimensions of reality, tend in the process of being narrated to become lifeless and petrified. Education is suffering from narration sickness.
>
> - Paulo Freire (1968, p. 57)

According to the National Council of Teachers of Mathematics (NCTM) standard of communication, “Instructional programs from pre-kindergarten through grade 12 should enable all students to...communicate their mathematical thinking coherently and clearly to peers, teachers, and others” and students need to learn “what is acceptable as evidence in mathematics” (NCTM, 2000, p. 60). But do teachers have a clear understanding of what is acceptable or do they believe that the only acceptable explanations are the ones that they themselves gave to the students? Can teachers accept alternative forms of explanation and methods of solution as mathematically accurate or do they want students to simply restate the teachers’ understandings of mathematics and the problem? The focus of this dissertation is the authoritative discourse practices of classroom teachers as they relate to individual students and large and small groups of students.

In many middle school mathematics classrooms, students are often asked to explain their understanding and reasoning of mathematical concepts and problems but

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This dissertation follows the style of the *Journal for Research in Mathematics Education*.

other teachers who do not share the responsibility for classroom explanations may be shortchanging students by not allowing them the experience that comes with equitable classroom discussions (Gergen, 1995; Hamm & Perry, 2002; Williams & Baxter, 1996). The Project 2061 *Habits of Mind* benchmark highlights the importance of equity in the dissemination of information, stating that understanding what others say and having others understand what you say is of equal importance (AAAS, 1993). Students who are asked, but not sincerely encouraged, to share their understanding and subsequently dismissed as irrelevant might be being taught that they have no authority in mathematical comprehension. Even among the most conscientious teachers, shared authority can be covertly, and overtly, denied if teachers require student participation in activities that the teacher has established and justified (Hamm & Perry, 2002). Teachers often condition students through both exercising overtly authoritative and oppressive discourse conventions that hinder full participation in mathematics discourse (Gergen, 1995; Sfard, 2001; Williams & Baxter, 1996). Klein (1999) argued that teachers privilege comfort, familiarity, and safety when choosing pedagogical strategies, oftentimes merely mimicking the more “traditional,” teacher-dominated instruction they may have experienced as students. One of the inherent tendencies of these more teacher-centered, traditional approaches to teaching, and of those that most contradict constructivist pedagogies promoted in more recent standards of mathematics (e.g., National Council of Teachers of Mathematics, 2000), is the "position[ing of] learners as ultimately dependent on [the] authoritative and all-knowing [teacher] (Klein, 1999, p. 87).

Teachers' behaviors are, in a large way, shaped by their beliefs (Borko & Putnam, 1996; Ernest, 1989; Lampert, 1990; Pajares, 1992; Schoenfeld, 1998; Strauss & Shiloney, 1994; Thompson, 1992). Teachers' beliefs about classroom discourse affect many of the moment to moment communicative decisions teachers make throughout the school day. I acknowledge that many other environmental factors, such as curriculum concerns, accountability issues, and student attitudes, influence teachers' discursive behaviors, but what a teacher believes about the nature of learning and teaching and the nature of knowing and doing mathematics affects the discourse strategies she chooses to employ in her classroom (Cazden, 2001).

Another aspect of teacher beliefs that likely influences decisions and actions in the classroom is the belief about where various aspects of authority lie as they pertain to classroom issues and the teaching and learning of mathematics. Who are these various authorities and where and how did they become authorities? Teachers' beliefs about the answers to these questions are important and influential to their teaching practices, including classroom discourse practices, and student learning.

For the duration of this study, I will denote teachers whose teaching practices are in line with the current mathematics education reform movement in the United States (cf. National Council of Teachers of Mathematics, 2000) as "reform oriented." For such reform oriented teachers, current educational reform in mathematics education serves as an important influence on how day-to-day and moment-to-moment mathematics classroom activities are facilitated.

Bakhtin describes authoritative discourse as a “privileged language that approaches us from without; it is distanced, taboo, and permits no play with its framing context” (1981, p. 424). Because of its very nature of Holiness and absoluteness, we are required to entirely accept the authoritative word, according to Bakhtin, as we cannot divide the authoritative word into parts to be accepted to various degrees. Bakhtin’s explanation of “authoritative discourse” described the “special” language used in many mathematics classrooms as “in lofty spheres, not those of familiar contact” (pp. 342-343). From these lofty spheres, many mathematics teachers end up “speaking down” to students in a language steeped with the authority of catechetical Latin – from the mathematical gods to the ears of the students through the voice of the teacher. However, Bakhtin does allow some variety in levels of contact of authoritative discourse to the receiver; “authoritative discourses may embody various contents: authority as such, or the authoritativeness of tradition, of generally acknowledged truths, of the official line and other similar authorities” (p. 344). What effects do these types of discursive conveyances of authority have on students and their understanding of mathematics? If the word of the teacher is taken as the sole mathematical authority, will students see themselves as having any contribution to the discourse?

What are the connections between this discussion of Bakhtin’s rhetoric and the teaching and learning of mathematics? One of the most crucial aspects of discourses are that they “have a social interactional aspect, with a basis in social relations of power; this regulates how positionings come about and how evaluations are made” (Morgan, Evans, & Tsatsaroni, 2002).

By the time students reach the middle grades most will have learned from previous teachers not only, mathematical facts and concepts, but also important attitudes concerning mathematical knowledge. Past teachers cultivate these attitudes about mathematics, to a large extent, through the modes of classroom discourse they enacted (Lampert, 1990). Students may have learned from their teachers that “mathematics is a discipline…to which they have little to contribute” (Hamm & Perry, 2002, p. 136). Conversely, these researchers found that those teachers who gave students the opportunity to “move beyond learning simple procedures” (p. 136) tended to produce students with expanded mathematical understandings and more of a sense of ownership of mathematical knowledge.

Williams and Baxter (1996) argued that most of the mathematical knowledge discussed in classrooms has already been created and attempts to make students feel as though they are architects in the construction of mathematics can be meaningless because these attempts are often as “ritualistic” as the known mathematics itself. “Emphasizing communication in a mathematics class helps shift the classroom from an environment in which students are totally dependent on the teacher to one in which students assume more responsibility for validating their own thinking” (National Council of Teachers of Mathematics, 1989, p. 79). For a teacher to establish classroom communication patterns whereby students share in authority, she must go beyond just asking questions (Carpenter & Lehrer, 1999). Rather, teachers’ reactions to students’ answers to questions go further to encourage or discourage future classroom communication than simply asking effective questions (Hamm & Perry, 2002; White, 2003). Hamm and Perry (2002) offer an example of an elementary grades teacher that seemed to be developing mathematical authority of students by actively engaging her students only to conclude the day's lesson by thanking them for being a "good audience" (p.136), a comment that overtly positioned the students as onlookers to the authoritative teacher.

Figure 1 shows the logic followed above linking teachers' beliefs and practices with the domain of mathematics, their knowledge of mathematics education reform, and two theories of learning. Figure 1 should be looked at not as all encompassing of all influences on teachers' beliefs and practices in the classroom. Instead Figure 1 should be considered as a small yet significant piece of teacher practices on which I have chosen to concentrate my work.

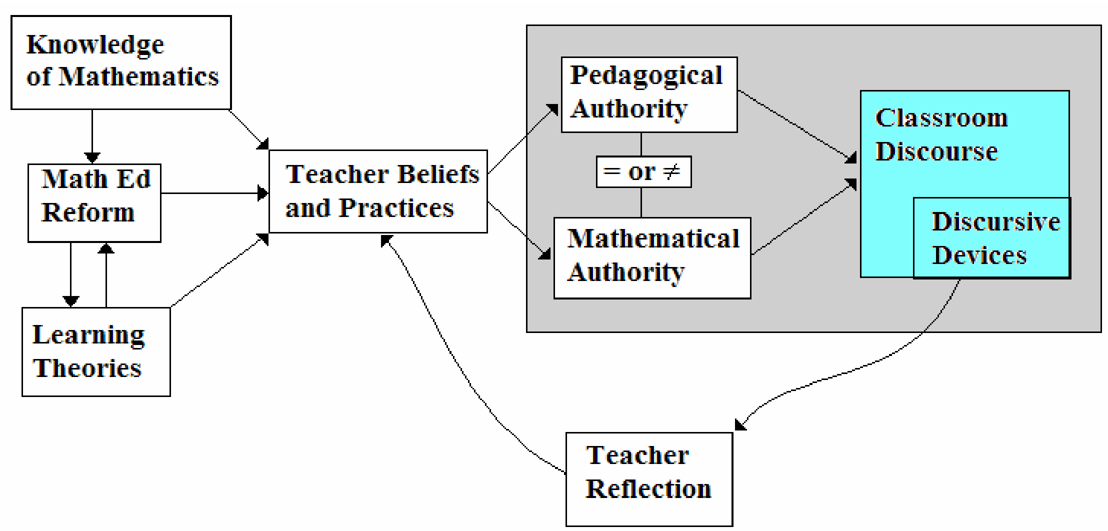

Figure 1. Logic Map.

The arrows in Figure 1 indicate proposed and possible directions of influence, based on previous research. For example, the arrow pointing from Learning Theories to Teachers' Beliefs and Practices indicates that teachers' beliefs and practices are likely influenced by what they know about theories of learning. Also, the large box that encompasses Pedagogical and Mathematical Authority, the two types of authorities present in the classroom, Classroom Discourse, and Discursive Devices is highlighted in gray to indicate that these will be the focus of this research. The equal and not equal signs indicate that teachers may or may not conflate these two types of classroom authority.

Statement of Purpose

The purpose of the study was to consider the ways teachers use subtle and not so subtle discursive devices to convey and share authority in their classroom. The research questions for this study were:

1. How do mathematics teachers' classroom discourse practices reflect their willingness to share mathematical authority amongst their students?
2. How do mathematics teachers' classroom discourse practices reflect their willingness to accept solution methods and explanations divergent with their own?

Definitions

For the purposes of this study, I have defined many of the terms used throughout this dissertation.

The term **authority**, as used throughout this dissertation, means **mathematical authority** and is not intended to be confused with classroom authority. When defining

mathematical authority, then, questions of and relationships to authorship and understanding arise (Hackelton, 2002). If a student has what she considers a new or original thought, then that student is considered as the author and, when coupled with an understanding of the associated mathematics, I consider her to have **mathematical authority**. Boyd-Batstone (2002), in the context of reader response theory, discusses the issue of authority and authorship this way:

> In Spanish, the word for power, poder, connotes not so much domination, but authority to know and to do - to have authority as in authorship. My use of the phrase ‘sharing power’ carries the idea of being an author of knowledge and action. Sharing power in the classroom means that knowledge and action are shared by the teacher and students as coauthors of the curriculum. Being powerless means passively accepting instruction as handed down from teacher to students. It is not authoring one’s ideas and actions; it is being acted upon. Sharing power in the classroom is sharing the authority with students to act as coauthors of their learning and their lives. (p. 133)

Mathematical agency is used in this dissertation in a similar way as mathematical authority. Mathematical agency is defined as critical agency in the structure of mathematics. A more thorough explanation of critical agency and the subtle differences between general critical agency and mathematical agency is found in the next chapter.

In order to clarify the discussion of discursive devices teachers use in their classrooms, it is necessary to define those devices that will be examined. **Cloze questions** or **cloze-type questions** originate in reading comprehension assessment. The cloze procedure is a test for reading ability where words are removed from a prose passage at regularly occurring intervals and students are to fill in the missing word(s). In this dissertation, **cloze** or **cloze-type questions** refer to questions that the teacher asks where he is looking for a specific word or phrase to “fill in” the omitted word or phrase. An example of a **cloze** or **cloze-type question** is “John, in fourths I should have . . .?,” where there was one accepted answer. An example of a less obvious, but appropriately categorized **cloze-type question** is “Using two words how I would change a decimal back into a fraction?,” where the only accepted answer was “cowboy rule,” a pneumonic device for converting fractions into decimals.

Echoing refers to repeating precisely what was previously uttered by the student. Echoing is often used in conjunction with cloze or cloze-type questions.

Revoicing is defined in this research to be the redecorating of a student’s response by the teacher (O’Connor & Michaels, 1993). **Revoicing** is similar to echoing in that part of a student’s utterance is repeated, but a revoiced response does not have to be the student’s response verbatim.

Overview of Research Methods

The research reported in subsequent chapters was a case study of an experienced middle grades teacher of an eighth grade mathematics class. The teacher was chosen for the study because of her highly interactive, reform-oriented style of teaching. From previous conversations and observations, I learned that she uses a variety of teaching methods including instructional technology and collaborative group work, among others. I observed the teacher’s classroom for an extended period of time collecting field notes, video data, curricular materials, and interview data. I transcribed all data and conducted a discourse analysis of classroom interactions and interviews based on identification and explanation of various discursive devices previously in the literature shown to be potentially authoritative. After identification of these devices in the interactions and interviews, I used the context of the interactions to contextualize each usage to inform a qualitative analysis of all collected data.

Significance and Implications of Study

While there is much research available on discourse in the mathematics classroom, there is not much on the uses of authoritative discourse in the middle grades mathematics classroom. As discourse analysis, as a methodology, is more like a heuristic, there is no one sure way to analyze the interactions in a given classroom that will necessarily work for another classroom (Lindsay, 1990). However, one of the strengths of this study is in furthering discourse analysis as a valid and informed research method.

Teachers that believe they use mathematics reform strategies and that conflate their classroom authority and their mathematical authority may not be practicing what they claim to preach. Teachers need to be self-reflective in order to be more effective (Tzur, 2001). This study was intended to incite teachers to reflect on their own practices of discourse and re-examine their use of authoritative discourse in the classroom.

Limitations and Delimitations

One of the delimitations of this study was in its intention. The intention of this research was to garner a critical and deep understanding of the practices of one teacher at one school in one school district and of a limited number of students. I do not hope to claim that my conclusions generalize to anyone but the participating teacher. It is my hope, however, that the methods of analysis, on which my conclusions are based, can be utilized to gain further understanding of teachers’ discourse practices by teachers and researchers. Although this study is limited in breadth of participants, there is a depth of understanding that should result from focusing attention on a limited number of teachers.

As a mathematics teacher, I am sympathetic to many of the pressures and constraints that public school teachers experience. While being sympathetic to these issues, I tried to maintain a critical perspective throughout the research process. This critical outlook was most difficult during interviews and when analyzing the observed discourse. During these processes, I resisted the temptation to answer affirmatively to the teacher's oft asked question, '*do you know what I mean?*' Rather, I probed for further clarifications and would continue to ask why, even when I could make assumptions about the teacher's meanings and motives.

CHAPTER II

THEORY AND LITERATURE REVIEW

Introduction

The purpose of this study is to examine teacher's discursive practices in mathematics classrooms and offer a mechanism for teacher reflection. Following the structure laid out in the first chapter, I will expound on the state of research on these various influences on and enactments of classroom communication. In the first section, I discuss the issues surrounding the current reform movement in mathematics education in the context of classroom communication. Next, I discuss the nature of mathematics and attempt to describe some epistemological aspects of mathematics. I then present a description of the facet of constructivism most prominent in the current mathematics reform movement and the importance of students taking ownership of mathematics. Within this section is a brief discussion of the theory of situated cognition and its relationship to constructivism. Included is a discussion of mathematical authority and agency. In the next section, I present Mikhail Bakhtin's rhetoric in terms of authoritative and internally persuasive discourse and how I view classroom interactions through the lens of dialogism. I then discuss classroom discourse and the nature in which it is used and suggest ways that classroom communication may be utilized to help students learn more and better mathematics. The next section describes the different types of authority present in the mathematics classroom and shows how these authorities can be used, misused, conflated, and shared. At the locus of the discussion in this chapter are discursive devices that teachers may use and their historical context. Finally, I discuss teacher reflection and its importance in the growth and maturity of a reform teacher.

Reform Mathematics

What is “reform” and what does it mean for a teacher to be using a “reform curriculum”? Schools and education have been in a state of reform since the time of Socrates (Nelson, Palonsky, & McCarthy, 2004). A standards-based reform movement is currently at the forefront of changes being made in several American public education systems. This most recent reform movement primarily stems from *A Nation at Risk* (National Commission on Excellence in Education, 1983), in which alarmist claims and bold, yet vague, recommendations were made in favor of improving the nation’s school system through a standards-based approach. Although the National Commission of Excellence in Education’s frantic call for standardization was well criticized (e.g., Berliner & Biddle, 1995), American educators began a new movement of standards-based reform that would seek to improve this nation’s public school system.

The primary goal of reform in mathematics education is improvement in the teaching of mathematics (Sfard, 2000). The National Council of Teachers of Mathematics (NCTM) assembled a commission to create a collection of standards for the teaching of mathematics that were grounded in the following assumptions: (1) “Teachers are key figures in changing the ways in which mathematics is taught and learned in schools”, and (2) “Such changes require that teachers have long-term support and adequate resources” (NCTM, 1991, p. 2). The standards that have emerged (NCTM, 1989, 2000) have had a lasting impact and provided a guide for mathematics teachers wanting to reform their own teaching practices.

The National Council of Teachers of Mathematics' *Curriculum and Evaluation Standards for School Mathematics* (1989) and the *Principles and Standards for School Mathematics* (2000), below referred to as *CESSM* and *PSSM*, have had considerable and lasting effects on the field of mathematics education. What follows is a discussion of the some of the most important effects of the *CESSM* and *PSSM* on policies, practices, and perspectives in mathematics education.

One of the most important lasting effects of the *CESSM* (NCTM, 1989) was providing the mathematics education community with a set of common standards on which to base teaching and professional development practices, as well as mathematics education research. Through the organizational development of NCTM and the subsequent publication of the *CESSM*, the mathematics education community became more unified in their goals and the ways in which those goals were met (Ross, McDougall, & Hogaboam-Gray, 2001).

A set of national standards provided individual states with a guide in which to base their state standards. The positive effects on state curriculum standards in states such as North Carolina (Joyner & Bright, 2001), Massachusetts (Riordan & Noyce, 2001), Missouri (Reys, Reys, Lapan, Holliday, & Wasman, 2003), and others is well documented.

Possibly the most important and most lasting effect of the *CESSM* was its effect on how the teaching and learning of mathematics was viewed. Readers and followers were to come to understand that, in order to do mathematics, it was necessary to understand mathematics. The emphasis on conceptual development of mathematics became and would remain the focused goal of mathematics teaching over a previous traditional emphasis on mastery of basic skills and computational algorithms.

A shifting perspective on how mathematics is learned and should, consequently, be taught was strengthened and accelerated by the release of the *CESSM*. Since the late 19^{th} century, objectivism and behaviorism have come and certainly seen a downturn in psychological practices to be replaced by cognitive approaches (Mills, 1999). As educational practices have held the hand of those in psychology, the likes of Pavlov, Thorndike, and Skinner have steered mainstream practices in education toward a behaviorist model that still guides many mathematics teachers.

Skinner claimed that, accepting the behaviorist model for teaching and learning, “as a mere reinforcing mechanism, the teacher is out of date” and can be effectively replaced by “mechanical and electrical devices” (1954, p. 94). Skinner suggested, as a solution to the problem of inefficiency, a teaching machine that aided the teacher in instruction and assessment. The notion of increasing efficiency in education through the use of machines to aid teachers has been extended by many to one of machines that replace the teacher or the development of teaching scripts so that the only qualifications for becoming a teacher would be literacy. Efficiency in education certainly appears a noble goal, save the fact that many of the widgets are not “manufactured” correctly and do not, therefore, maximize their potential. The efficiency model of education is, thus, very inefficient for many students. The educational philosophy expressed in the *CESSM* countered a behaviorist, efficiency philosophy of education with goals of teaching mathematics to all students with practices that were supported by constructivism and cognitive psychology.

In 2000, NCTM introduced the revision to the *CESSM*, titled *PSSM*. The revision was the culmination of years of reflection on the ways in which the original document had been viewed and utilized. The revision amended several aspects of the original document and accentuated some less emphasized factors. I discuss some of the differences below between the original and the revised version of the NCTM *PSSM*.

The *PSSM* document is clearly the result of many years of reflection on how teachers and policy makers, as well as mathematics education researchers, have tried to use and follow the original standards. The revision is a more eloquent document, condensing the standards for each grade band into a set of standards that apply to every grade level though the standards are to be applied differently for each grade band.

Although the NCTM (2000) standards for the teaching and learning of mathematics are the most prominent, they are not the only set of standards teachers are being asked to follow. There currently exist a multitude of supplementary and alternative standards by which teachers can or are required to abide (American Association for the Advancement of Science, 1993; National Board for Professional Teaching Standards, 2004; Texas Education Agency, 2001).

The National Board for Professional Teaching Standards' *Ways of Thinking Mathematically* standard for adolescence and young adulthood (National Board for Professional Teaching Standards, 2004) asserts "Accomplished mathematics teachers develop students' abilities…to justify and communicate their conclusions, and to question and extend those conclusions.” But how are teachers supposed to follow reform standards when the standards give no explicit directions? The NCTM *Professional Standards for Teaching Mathematics* (1991) at least provide clearer directions as to the ways teachers can follow the standard of classroom discourse. The role of the teacher in classroom discourse is made relatively explicit through such suggestions as

> the teacher of mathematics should orchestrate discourse by…listening carefully to students’ ideas; asking students to clarify and justify their ideas orally and in writing; deciding what to pursue in depth from among the ideas that students bring up during a discussion;…deciding when to provide information…and when to let a student struggle with a difficulty. (NCTM, 1991, p. 35)

NCTM’s communication standard states that “Instructional programs from pre-kindergarten through grade 12 should enable all students to . . . communicate their mathematical thinking coherently and clearly to peers, teachers, and others” and students need to learn “what is acceptable as evidence in mathematics” (NCTM, 2000, p. 60). How do teachers know if all students have been enabled to communicate “coherently and clearly”? What are the ways by which a teacher determines what is acceptable as evidence? Yackel (2001) suggests that perhaps teachers and texts not be deterministic, focusing only on conclusions, but focusing more on reasoning and encouraging the use of “group thinking,” meaning a shared reasoning process whereby students interact to form mathematical validation in various size groups. Teachers should also serve as facilitators in students’ reflective discourse practices by initiating discussions and taking a non-participatory evaluative role (Cobb, Boufi, McClain, & Whitnack, 1997). A long-standing approach to classroom discussions is the Initiation-Response-Follow-up (IRF) sequence where the teacher initiates the conversation, typically with a question, a student responds to the initiation, and the teacher serves as the evaluator or the student's response (Wells, 1993). However, some reform efforts in mathematics education (e.g., NCTM, 1991; 2000) have suggested that an approach to classroom discourse that situates the teacher as an orchestrator of student interactions may give students a more active role in explaining and learning mathematics (Forman & Ansell, 2001). Empson (2003) asserts that many educators do little more than give lip service to the ideas of reform-based teaching when teaching low-achieving students but that through long-term exposure to positive interactions, these same students could have developed their "mathematical agency" (p. 340).

The suggestions and findings reported above show that there exists much concern for the ways teachers position themselves in the classroom relative to students during mathematical discussions. But teachers have an obligation for at least some evaluation of what their students know and can do. Determining what students can do mathematically is not unproblematic. However, determining what students know or how think about mathematics raises more philosophical questions about teachers' beliefs.

Knowing Mathematics

What does it mean to "know" mathematics or think mathematically? How do teachers know that their students know mathematics or can think mathematically? Contrary to the unfortunate beliefs of many students and teachers alike, "mathematics is not something that was handed down to some mathematical 'Moses' in by gone times. Mathematics is something that man himself creates…" (Wilder, 1968, p. 4). As is the case with most domains of study, the development of mathematics did not and does not occur without strong and profound cultural and societal influences (Wilder, 1968).

One of humans' characteristics that distinguish us from other animals is our ability to represent the world symbolically (Wilder, 1968). "Man possesses what we might call symbolic initiative; that is, he can assign symbols to stand for objects or ideas, set up relationships between them, and operate with them on a conceptual level" (p. 5). This differs from "symbolic reflex" behavior, which is a programmed behavior stemming from conditioned responses to symbols, as with memorization of arithmetic rules for fractions without an accompanying conceptual understanding of what it means to, for instance, divide by a fraction or to press a sequence of buttons in order to receive a food pellet. Wilder asserts that all too often, teaching of mathematics involves an emphasis on symbolic reflex behavior rather than symbolic initiative.

Adding It Up (National Research Council, 2001) discusses the recent and rapid evolution of what it means to know mathematics and how this knowledge is to be recognized in students. The authors provide five interwoven "strands" of mathematical proficiency: conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition. Conceptual understanding reveals itself in students in ways such as a student's ability to explain why particular relationships hold in a problem and why certain operations are used in a problem. Procedural fluency is self-explanatory with additional consideration given to flexibility, efficiency, accuracy, and appropriateness in performing mathematical procedures. Strategic competence is measured by an ability to "formulate, represent, and solve" (p. 116) given problems.

Adaptive reasoning is an ability to reason logically about a problem and reflect on, explain, and justify the solution. Productive disposition is revealed as a "habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy" (p. 116).

Constructivism and Situated Learning

Jean Piaget is probably best known for his stage theory of cognitive development, but what many learning theorists overlooked in Piaget's work, until the theory of constructivism was widely accepted, was his intended use of construction and the ways it described constructivism (von Glasersfeld, 1996). Constructivism is a theory of learning that, at its essence, is based on the individual's experiences and how the individual builds knowledge on prior knowledge based on experiences. These experiences involve the learners interacting with their environment (Janvier, 1996), which can include their physical surroundings as well as their peers, thus the social aspect of learning. With regard to the influences of social interaction on knowledge construction, von Glasersfeld argues that "because others add up to a major part of the individual's experiential environment, they will have a considerable role in determining which behaviors, concepts, and theories are considered 'viable' in the individual's physical and linguistic interactions with them" (1996, p. 309).

Another important figure in the development of constructivism was Lev Vygotsky. Vygotsky's (1978) constructivism utilizes more of the social aspects of learning. Vygotsky is probably best known for his "zone of proximal development," (ZPD). In the words of Vygotsky, the ZPD is defined as "the distance between the actual development level…and the level of potential development…under adult guidance or in collaboration with more capable peers" (p. 86).

Vygotsky's strong emphasis on the social influences of learning has caused some to disagree on the relationship of Vygotsky and constructivism, or at least Piagetian constructivism (cf. Lerman, 2000a; Steffe & Thompson, 2000). Lerman (2000a) proposes that the theories of Vygotsky and Piaget actually contradict. Steffe and Thompson (2000), in defense of (radical) constructivism, suppose that Piaget's ideas on social interaction were much in line with those of Vygotsky. Some of the commonalities between Piaget and Vygotsky were the beliefs that social factors were important for development, internalization is a process of transformation, and that the goal of development was the individual's understanding (DeVries, 2000).

Constructivism is a theory of learning, not a theory of pedagogy and that teaching is, by its very nature, an intervention and "a-constructive" process (Janvier, 1996). Von Glasersfeld (1996) admits that whereas constructivism is not a guide as to what to do to teach effectively, it is a guide of what not to do. One example of a common a-constructivist pedagogical practice is labeling of students' naïve conceptions as misconceptions and then trying to replace them with the conception deemed proper by some "expert" of mathematics. Another a-constructivist practice that teachers commonly employ is placing an unwarranted emphasis on the symbols of mathematics. Glasersfeld is adamant about the importance relating the meaning and conceptions that these sometimes arbitrary symbols only represent.

Both proponents of constructivism and those of a situated learning theory assert that Vygotsky was an early and important influence in the development of their respective learning theories. Situated learning proponents are particularly interested in Vygotsky's Zone of Proximal Development (ZPD) (cf. Lerman, 2000a). Of special importance in Vygotsky's (1978) description of the ZPD is the emphasis on adult guidance and peer collaboration with those more capable. These defining elements of the ZPD support a situative perspective that learning occurs in a community of practice.

Situated learning is a theory of learning whose major concern is learning in relation to social practices. Lerman (2000a) suggests that the "social turn" in mathematics education occurred somewhere around 1988, with the coinciding publications of Bishop's (1988) *Mathematical Enculturation,* Lave's (1988) *Cognition in Practice: Mind, Mathematics, and Culture in Everyday Life*, and Walkerdine's (1988) *The Mastery of Reason*. These publications helped bring to the forefront, a "new" way of thinking about mathematical learning.

Lave (1988) asserts that learning occurs in the context of cultural activities and social interactions, where learners become a part of a "community of practice." Within these communities of practice, members have certain shared beliefs and practices that define membership. With respect to the production and nature of knowledge under the theory of situated learning, "activities of person and environment are viewed as parts of a mutually constructed whole" and not as separate entities" (Bredo, 1994, p. 23). Brown, Collins, and Duguid (1989) assert that "knowledge is situated, being in part a product of the activity, context, and culture in which it is developed and used" (p. 32). Peressini, Borko, Romagnano, Knuth, & Willis (2004) concur and add that through a situative perspective, what is learned is not just content but important situational and contextual understandings.

The ways learners are enculturated into various communities of practice is through a sort of apprenticeship process Lave and Wenger (1991) call "legitimate peripheral participation." Brown and colleagues (1989) discuss the process as a cognitive apprenticeship.

Some of the differences and commonalities between these learning theories have been previously discussed. I will now touch on some others. Peressini and colleagues (2004) propose that one of the differences between constructivism and situated learning theories is the notion of *transfer*. Constructivists contend that, because knowledge can be gained independently of the context, this knowledge is decontextualized and can be transferred to other settings than the one in which it was originally acquired, because the context is not important. Because a situative view of learning is concerned with the practices in which individuals have learned to participate and "consider individuals' acquisition and use of knowledge as aspects of their participation in social practices" (Greeno, 2003, p. 315), a question of transfer is likely inappropriate (Peressini et al., 2004). Instead of the transfer of knowledge across settings, researchers from a situative perspective would be more interested in "consistency of patterns of participation across situations, conditions under which successful participation in activity in one type of situation facilitates successful participation in other types of situations, and the process of *recontextualizing* resources and discourses in new situations” (Perissini et al., 2004, p. 70, emphasis in original).

Perissini and colleagues (2004) also contend that another difference between cognitive perspectives, including constructivism, and situative perspectives is the unit of analysis for the research of learning. Cognitive perspectives focus on the individual learner as their unit of analysis and, according to Lave (1988), the unit of analysis, from situative research perspective, is the social group or activity. Cobb and Bowers (1999), disagree that this distinction is a necessity and that the choice of unit of analysis depends on the purpose of the research.

The starkest difference between constructivism and situated learning is the focus of activity. Traditional cognitive perspectives, such as those of constructivists, understand learning as an individual phenomenon (Borko et al., 2000). The learner’s environment, such as the social situation, the teacher, the classroom, the text, may be considered to have little impact on the students’ acquisition of knowledge and understanding of mathematics. What has the most impact on learning are the learner’s prior knowledge and the ways that new information is adapted into the mind of the learner. Borko and colleagues contend that, from this perspective, knowledge is considered to be decontextualized and can be transferable from one domain to another.

One of the clearest commonalities between constructivism and situated learning is that both theories require action from the learner. Both theories and their assorted varieties posit active involvement as a necessary requirement for successful learning. Unlike behaviorism, which requires physical action in the form of practice for the mastery of skills, constructivism and situated learning require action which is more on a cognitive level.

Another of the commonalities of both theories involves students' sense of ownership of mathematics. Both situated learning theorists and constructivists consider ownership as a necessity for understanding. In order for students to understand mathematics, they must take ownership of the ideas being taught (Carpenter & Lehrer, 1999; Knuth, 2001; NCTM, 1991, 2000; Prevost, 1996; Sfard, 2000). Students must begin to rely less on the authority of their teachers to tell them if they have the correct answer or if they are solving the problem in the correct way (Prevost, 1996). Von Glasersfeld (1995) argues that in order "to solve a problem, one must first see it as one's own problem" (p. 14).

I am not arguing that the teacher is to be removed as a mathematical authority, which means, for the purposes of this dissertation, the authority to know, do, and create mathematics, but that the authority to confirm correctness and appropriateness develop within each student under the guidance of the teacher. To restrict the teacher from any intervention in the learning process would be a misinterpretation of the "profound constructivist principles underlying the current reform movement" (Sfard, 2000, p.185). Instead of completely disregarding the importance of the teacher's authority to know mathematics, Prevost (1996) suggests that the focus is placed on how a student's "understanding of mathematical concepts grows through his or her participation in well-chosen activities, in discussion with the teacher and fellow students, and through reflection" (p. 51).

From a constructivist paradigm, teachers do not construct students' knowledge. Students' knowledge construction is the role of the students. Teachers should take a facilitative role in classroom interactions instead of a domineering one (Cobb et al., 1997; Hamm & Perry, 2002; Lampert, 1990; National Research Council, 2001; Sfard, 2000). Teachers should be available to students as a resource but should not be relied upon as the only mathematical authority in the classroom (National Research Council, 2001; O'Connor, 2001). Teachers have to decide when to give solutions or explanations and when to let students do the answering and explaining (National Research Council, 2001). Hatano (1996) suggests, as one of his principles of mathematics education, that teachers should enable development of student autonomy in judging the validity of their own and fellow student's mathematical explanations and solutions and that these criteria not be imposed upon the students based on the authority of the teacher.

In a comparison of the teaching of mathematics between elementary level classrooms in Japan and America, Stigler, Fernandez, and Yoshida (1996) note Japanese teachers' reluctance to base evaluation and justification of mathematics on their own or an outside authority, such as the text or the domain of mathematics. The Japanese teachers they observed, instead, deferred judgment to the mathematical authority of the "community" of students. In contrast, Stigler et al. found the American teachers they observed to frequently rely on various mathematical authorities including themselves and an ambiguous "they."

What are the ways teachers can foster ownership of mathematical knowledge among students? Prevost (1996) provides an appropriate example of students constructing their own understanding of mathematical relationships and concepts in the context of teaching multiplication of two digit numbers. When Prevost let the students discover their own methods of multiplying 27 by 2 and 27 by 4, “a host of ideas came up, including what we would call the ‘traditional algorithm’” (p. 50). Prevost then followed the discovery with the necessary discussions of students’ discoveries. These discoveries that may or may not be divergent from those taught be a teacher or a textbook make up a significant part of what I am calling “mathematical agency.” Below, I discuss the origins, definition, and examples of mathematical agency.

Mathematical Agency

In order to be more authentically scientific, and certainly mathematical, students should respect, but also challenge, “socially established authorities” (Shotter, 1995, p.46), including the teacher. When students exercise their agency in the domain of mathematics, which I am calling *mathematical agency*, they are embodying this challenge.

Why *mathematical agency*? Is the notion of agency important to mathematics learning? The term, *mathematical agency,* is a term not without need for explanation. I will begin this explanation by discussing origins and definitions of the term *agency*, how the term has developed, how it has been used in research. I will then demonstrate how *agency* is extended to the field of school mathematics as *mathematical agency* and the ways notions of agency and mathematical agency can inform research and teaching practices. I will conclude with some suggestions on the ways in which questions concerning agency and mathematical agency have been previously investigated.

A discussion of agency is likely not to occur without some mention of at least one of the fields of sociology, philosophy, history, psychology, social psychology, anthropology, or linguistics, among others. However, for the purposes of this discussion, it would be helpful for the reader to consider that, for the most part, I will try to keep the concept of agency and it's development on a plane that is pragmatic and "researchable," providing answers to practical questions, e.g., *what does agency look like?* or *how will I know it when I see it?*

Agency, like *identity* and *self*, is a term or, better, concept that has roots in sociology, anthropology, psychology, social psychology, and cultural studies (Holland, Skinner, Lachiotte & Cain, 1998) and is developed in and through social actions and interactions. As scholars and researchers do not necessarily agree on one definition of agency to meet all their research or academic needs, they operationally define agency as it best suits their purposes.

The concept of agency can be traced back to John Locke's denial of the power of tradition (Emirbayer & Mische, 1998). With this rejection, "a new conception of agency emerged that affirmed the capacity of human beings to shape the circumstances in which they live" (p. 965). Subsequent social theorists such as Adam Smith, Jeremy Bentham, and John Stuart Mill continued to equate agency with a sense of deliberate action promoting "freedom and progress" (p. 965). Other notions of agency and freedom took markedly different turns through the writings of other scholars of Locke's time such as Rousseau and Kant. Kant divided reality into two opposing and mutually exclusive orders: the conditional and the normative. The latter described Kant's notion of freedom, "as normatively grounded individual will, governed by the categorical imperative rather than by material necessity (or interest)" (Emirbayer & Mische, 1998, p. 965).

Below, I illustrate several of the ways that scholars have defined agency and follow up with a synthesis of the definitions and explanations. The first, and possibly the least easily traversed, of the many definitions of agency combines Emirbayer and Mische's (1998) notion of the interplay between the relative constituents of time: the past, present, and future. They define agency as

> the temporally constructed engagement by actors of different structural environments—the temporal relational contexts of action—which, through the interplay of habit, imagination, and judgment, both reproduces and transforms those structures in interactive response to the problems posed by changing historical situations. (p. 970)

Another definition of agency comes from Gramsci (1971) who defines critical agency as intentional action that defies hegemonic practices which guarantee and validate social domination of certain individuals by others.

Inden (1990) defines agency as

> the realized capacity of people to act upon their world and not only to know about or give personal or intersubjective significance to it. That capacity is the power of people to act purposively and reflectively, in more or less complex interrelationships with one another, to reiterate and remake the world in which they live, in circumstances where they may consider different courses of action possible and desirable, though not necessarily from the same point of view. (1990, p. 23)

Similarly, Pruyn (1999) defines agency as "purposeful action taken by an individual, or group of individuals, in order to bring about change" (p. 20).

Gutstein (2003) asserts that students need to become involved in social injustice issues and, in order to do so, they must develop a sense of agency, which he defines as "a belief in themselves as people who can make a difference in the world, as ones who are makers of history" (2003, p. 40). Gutstein uses *agency* in the tradition of Freire, as intentionality – as conscious determination of history.

As the definitions have become increasingly intuitive and concise, I will elaborate on my definition and theory of choice. I prefer Giddens' (1994) theory of structuration and of agency as a theory of action and practice. Some explanation of Giddens' structuration theory is necessary.

Giddens' (1994) structuration theory proposes that there exist social structures, not to be thought of as rigid, unchanging, and impenetrable structures, like the walls of a building or tunnel, but, instead, that are both constraining and enabling. Analogously speaking, the contract between a record company and a musical artist would serve to both enable and constrain both parties of the contract. Giddens (1976) equates agency and action and describes agency contextually. He describes as a "necessary feature of action that…the agent *could have acted otherwise*" (p. 56). Giddens proposes that agency and structure work together from opposing sides. Structure can both constrain and enable agency and agency, through actions, can reshape and change structure.

Beaz (2000a) recognizes the Giddens' agency/structure relationship, but asserts a dichotomy between the two in the context of the perpetuation of racism. Baez relates the notions of agency (individuals) and structure (institutions) to power and claims that power is either exerted agentively, by an individual, or structurally, by an institution.

Baez (2000a) redefines agency in the context of the tenure and promotion process for faculty of color, "not as free will, but as *actions that are possible within the context of disciplinary power*" (italics in original, p. 385).

So what does agency look like? I present the following examples from the milieus of popular culture and school accountability. The first example of enacted agency surrounds the beginnings of what is now known as *gangsta rap* in the late 1980s. One of the groups that can be considered originators of this genre of music was *Niggaz with Attitude* (N.W.A.). The group's first record, titled *Straight Outta Compton*, contained songs with graphic and explicit lyrics, e.g., the song *F*** the Police*. The music and subsequent music videos of N.W.A. were considered controversial by most and unacceptable by the mainstream music and music video outlets of the time.

N.W.A. was likely well aware of the structure in which they were working. The structural constraints came from religious leaders, concerned parents, law enforcement, the Federal Communications Commission, and therefore, the mainstream media outlets. N.W.A. consciously worked against those constraints and, through the course of their actions, they altered the structure that constrained them. Through the publicity of negative media attention, the record's sales were high and, as a consequence of the realism and originality they brought to rap music, their influences would continue into the present forms of the hip hop or rap culture. The actions of N.W.A. in the late 1980s is what agency can, and often does, look like.

Other examples of enacted agency are given by Susan Ohanian's (2001) ongoing list of resistors to standardized tests and testing policies across the country. She provides readers with examples of actions from parents, students, teachers, administrators, and others concerned with the structure of standardized testing in their state. Ohanian describes parents who refuse to allow their potentially high scoring children to be tested by keeping them home from school on the days the test is administered. Ohanian also offers, as another example of agency through resistance, Steve Orel, who after being fired from his teaching position, opened up his own school in Alabama. Orel was fired for questioning why a Birmingham school “administratively” withdrew 522 students who were at risk of lowering the schools state test passing rate and allowed the state to take over their school. The action that was agentive, in this case, was the opening up of a new school. Orel worked within the system to change the system. This is agency.

Mathematical agency and agency, in the context of mathematics education, are terms heard with increased frequency in recent years, particularly in educational research dealing with identity (Boaler, 2002; Boaler & Greeno, 2000; Empson, 2003) and equity or social justice (Gutstein, 2003). Different mathematics education researchers define agency and mathematical agency as it best suits their research agendas. We are fortunate when a researcher that chooses to use highly contested and contextualized terms, such as mathematical agency, operationally defines these terms. Often, however, we are not so fortunate, e.g., Empson’s (2003) vague and casual use of the term *mathematical agency*. Although mathematical agency was not Empson’s focus, she concludes with suggestions on ways students can aid in “the development of [their] mathematical agency” (p. 340). We are subsequently left to wonder what mathematical agency is and how it relates to other notions of agency.

In Empson's defense, in a previously published work, she presented a detailed description of student agency and the need for its cultivation in mathematics classrooms (2002). Empson's casual use of the term mathematical agency may have implications that are important to the current discussion. Boaler and Greeno (2000) follow the combined examples of Empson (2002; 2003), by using and researching the concept of agency in the context of mathematics education. Boaler and Greeno also discuss Pickering's (1995) notion of the *agency of a discipline*, e.g., the agency of coordinate geometry. This leads to an important question. Is the notion of mathematical agency simply agency clothed in mathematics? In what follows, I hope to shed some light on this question.

Mathematical agency can and does exist at various levels, whether the agent is a teacher, student, or any person outside the immediacy of school mathematics. For the purposes of this discussion, however, I will constrain my examples and explanations to student mathematical agency as opposed to teacher agency. Mathematics classrooms are, in some ways, no different than other school structures. Within each mathematics classroom, there exist similar structural components as in any other classroom, e.g., rules, responsibilities, social norms, etc. In mathematics classrooms there are also crucial differences that distinguish them from other school classrooms.

The nature of traditional mathematics teaching and learning, which is the predominate model, despite recent reform efforts (NCTM, 2000), is based on customary rules and algorithms that are taught by rote and learned through practice and memorization. Even in standards-based reform mathematics classrooms, the structure as defined by classroom rules and social norms is supplemented with a mathematical structure defined by rules of mathematics and sociomathematical norms (cf. Cobb & Bauersfeld, 1998).

The imposition of mathematical structure is provided and reiterated through the actions of the teacher making her actually part of the structure that enables and constrains students. An alternative is that the teacher be a mediator for students between mathematical structure and the agency that is contextually defined in light of and in opposition to the constraints of the structure provided by classroom mathematics, thus *mathematical agency*.

Having defined mathematical agency relatively and in context, I will answer the question of recognition through some examples and suggestions. What does mathematical agency look like and how will I know it when I see it? Many of the unarticulated mental math operations that students invent or create can be considered examples of enacted mathematical agency. Traditional teaching of standard computational algorithms for double digit addition involves practice with “carrying.” The rationale behind carrying involves regrouping or, more precisely, the associative property of addition. A traditional mathematics teacher may have students practice with a set of exercises, encouraging mastery of the standard computational algorithm. Suppose a student decides that there must be an easier way to add, for example, 76 and 35 by regrouping in tens, saving what was not used to make another multiple of ten, adding the multiples of ten, and then adding what was left over to the previous sum. To clarify this idea, consider that the student might take 4 and 1 from the 35 leaving 30 and add the 4 to the 76 to get 80. Then the student would add the 30 and the 80 to get 110 and then add the left over 1 to get 111. This is a legitimate method of adding two digit integers and, for this student, may be a “better” way than was previously taught.

To be clear, if a teacher instructs the class or individuals on the use of computational methods that differ from standard algorithms, the use of the “alternative” methods is not necessarily agentive. The characteristic that makes these actions agentive is that the student was taught a particular method and, when asked to perform the computations, acted otherwise. Also notice that the structure of mathematics and the classroom constrained and enabled the student to act differently than expected.

Klein (1999) provides another example of student agency. She considered ways in which agency was not cultivated with preservice mathematics teachers during their teacher education programs. Klein found that the university students, i.e., the preservice teachers, felt disempowered and constricted by their mathematics experiences throughout college. Klein suggests that through experiences that provide opportunities for inquiry and exploration, students can develop identities as learners of mathematics. These identities, as well as providing a chance to make explicit their location in the discourse of school mathematics, can encourage preservice teachers to situate themselves within the structure and discourse of school mathematics and use this position agentively to change their own understanding of mathematics and how they might teach to support agency in their future students.

A non-example of mathematical agency, as I am using the term in this study, comes from the work of Gutstein (2003). In exploring questions of social justice and the cultivation of agency and identity, Gutstein worked with his seventh-grade mathematics students using a curriculum created to illuminate problems that were perpetuated in and by school and society and, in particular, inner city Chicago. Through his teaching and use of his curriculum, Gutstein showed ways that agency can be developed and supported in the context of a mathematics classroom. Gutstein's goals were for his students to use mathematics to act agentively, not to act agentively to learn mathematics. Although these two goals have much in common, they are also very different. The latter of the goals is what I here call mathematical agency.

These examples and non-examples also serve as suggestions for teachers interested in cultivating mathematical agency. Teachers supporting discovery of legitimate alternative computational and problem solving methods, not necessarily the ones taught in the text, can aid in supporting student agency. Also, when students work with teachers or administrators to help determine curriculum decisions, agency can be supported. When teachers serve as enablers for students to resist and change the various constraints, such as school policies or standard practices in mathematics, the teacher is cultivating students' enactment of agency in creative and positive ways.

I want to stress that, although the historical significance of each of the examples of agency I have provided is quite different, the acts had significance to each of the agents in each of the examples. Certainly Rosa Parks' agentive refusal to move to the back of the bus was historically significant to many future minorities and the civil rights movement of the 1960s, but that does not diminish the consequences of the agency of the student that discovers a way of doing things in mathematics differently than her teacher.

Authoritative Discourse

Mikhail Bakhtin and others in the Bakhtin circle, including Medvedev and Voloshinov, developed their "situational model of language that accentuates the social and concrete character of practical speech 'acts'" (Klancher, 1989, p. 84) as a response to the dogmatic language theories of Saussure and Stalin. From a critical perspective of discursive practices, one that views a non-unified or conflicting usage of language rather than one common language, Bakhtin provides the sociolinguistic notions of dialogic rhetoric and heteroglossia. Bakhtin uses the term "heteroglossia" to mean the state of conflict between centralizing and decentralizing forces in language that contextually determine its meaning. The notion of "dialogism," as explained by Bakhtin (1981)

> the characteristic epistemological mode of a world dominated by heteroglossia. Everything means, is understood, as part of a greater whole – there is a constant interaction between meanings, all of which have the potential of conditioning others. Which will affect the other, how it will do so and in what degree is what is actually settled at the moment of utterance. This dialogic imperative, mandated by the pre-existence of the language world relative to any of its current inhabitants, insures that there can be no actual monologue. One may, like a primitive tribe that knows only its own limits, be deluded into thinking there is one language, or one may, as grammarians, certain political figures and normative framers of 'literary languages' do, seek in a sophisticated way to achieve a unitary language. In both cases the unitariness [*sic*] is relative to the overpowering force of heteroglossia, and thus dialogism. (p. 426)

It is through the lens of dialogism that this research views classroom interactions.

Bakhtin describes authoritative discourse as a "privileged language that approaches us from without; it is distanced, taboo, and permits no play with its framing context" (1981, p. 424). Because of its very nature of holiness and absoluteness, we are required to entirely accept the authoritative word, according to Bakhtin, as we cannot divide the authoritative word into parts to be accepted to various degrees. However, Bakhtin does allow some variety in levels of contact of authoritative discourse to the receiver; "authoritative discourses may embody various contents: authority as such, or the authoritativeness of tradition, of generally acknowledged truths, of the official line and other similar authorities" (1981, p. 344).

For Bakhtin (1981), discourses can be divided into the almost, but not entirely, mutually exclusive categories of authoritative and internally persuasive discourses. Bakhtin posits authoritative discourse in conflict with and opposition to internally persuasive discourse. Internally persuasive discourse is a restating of text after personal internalization and interpretation. Bakhtin admits that it can happen, though rarely, that a discourse can simultaneously be authoritative and internally persuasive. In most cases, however, there is a diametrical opposition between these two forms of language.

What are the connections between this discussion of Bakhtin's rhetoric and the teaching and learning of mathematics? One of the most crucial aspects of discourse are that they "have a social interactional aspect, with a basis in social relations of power; this regulates how positionings come about and how evaluations are made" (Morgan, Evans, & Tsatsaroni, 2002). I suggest through this study that both authoritative and internally persuasive discourses are omnipresent in any mathematics classrooms that utilize some form of communication. In particular, I focus on the ways that various pedagogical practices, specifically discursive practices, reveal the presence of authoritative discourses. The application of the theories of Bakhtin to mathematics education, while not novel, is relatively uncommon (e.g., Forman, McCormick, & Donato, 1998b; Sfard, 2000).

Classroom Communication and Discourse

The connection between students' ownership of mathematics and classroom communication is the teacher. Carpenter and Lehrer (1999) considered teachers' classroom communication practices vital to fostering students' sense of ownership of the mathematics being communicated. Williams and Baxter (1996) assert that teachers and students belong to different societal groups and that one of the primary obstacles for teachers is how best to minimize these differences by fostering a dual student membership in the different groups. Legitimate student participation in mathematical discussions requires that the student first learn how to use the language of classroom discourse (Zevenbergen, 2000).

"Teachers at all levels…have to rely on the use of language, and textbooks cannot do without it. Yet…few language users have given much thought to the question how linguistic communication is supposed to work" (von Glasersfeld, 1996, p. 310). Brown and Yule (1983) categorize language functions as transactional and interactional. Lotman (1988) further describes the "functional dualism" of pedagogic discourse as to "convey meaning adequately, and to generate new meaning" (p. 34). Many mathematics teachers can use discourse to convey meaning, while under the guise of generating meaning. Thus, using the language of Wertsch and Toma (1995), although many teachers' classroom discourse may seem *dialogic* (i.e., to generate meaning) in nature, it is really *univocal* (i.e., to convey meaning). In this case, the voice of the teacher is the one that is heard regardless of who is doing the talking. Van Oers (2001) discusses his finding on the transmissional nature of classroom discourse in many mathematics classrooms

> As mathematical knowledge is assumed to be constituted of fixed entities, it is also believed that the elements of mathematical knowledge can be transmitted to children. The main communicational style of this approach follows the sender-receiver model that states that direct instructive language is needed to prescribe for children what to do with numbers. This point of view inevitably implies a special authoritarian relationship of a teacher towards his pupils. The teacher (as the one who knows) transmits pieces of mathematical knowledge to pupils (who don't know yet). Public discourse on mathematics in schools still follows mostly this point of view. (p.62)

Forman, McCormick, and Donato (1998b) studied the patterns of classroom communication during a lesson on algebraic patterns in an urban middle school in the first year of an educational reform project intending to help teachers cultivate student mathematical authority. In this study, Forman et al. demonstrate the ways even well-meaning, "reformed" teachers can unwittingly privilege… discourse that is "implicitly transmissional and authoritative in nature" (p.333). Knuth (2001) admits that both types of discourses can be appropriate depending on classroom factors that influence instructional goals but when teachers exclusively determine the goals of the classroom, the discourse becomes more univocal and of a transmissional nature, whereas when the students help to determine the direction of the classroom goals, the discourse becomes dialogic and the students share in mathematical authority. Classroom discourse, however, tends to lose importance for students if the intention of teachers is only to transmit knowledge of mathematics (Arlo & Skovsmose, 1998; Forman et al., 1998b). Authoritative discourse not only consists of the talk that occurs in classrooms, but also carries meanings that convey "subtle and not so subtle meanings about power" (Wink, 2000, p. 50).

Authorities in the Classroom

It is necessary to distinguish between different types of discursive authority common to classrooms and demonstrate the ways teachers cultivate and obviate these discursive authorities. In discussing issues of authority in the classroom, Freire (1968) "described traditional teaching as being dominated by a 'banking' concept of education" (p.58), whereby teachers merely deposit knowledge into the "empty vessels" that are the students. This view of teaching employs an attitude that there is no difference between a teacher's "professional authority" and "the authority of knowledge" (Freire, 1968, p. 59). Bourdieu and Passeron (1990) claim that teachers often exercise a mode of authority that is most focused on reinforcing itself through its own practice and called it arbitrary pedagogic authority. Frankenstein (1987) argues that when students supported the teacher's position as an expert by having only the teacher validate their answers, there was a potentially dangerous conflation of teachers' classroom authority with their mathematical authority. This conflation of authorities, argues Frankenstein, facilitates "the game," whereby students learn that, in order to achieve success, the rules of the game of school and mathematics, i.e., that the teacher is always "right" must be followed.

Zevebergen (2000) shows that when students challenge teachers' mathematical authority, the teacher may exercise classroom authority by shifting from classroom discussion to individual work, thus maintaining control over the classroom and denying students the opportunity to publicly share and socially construct "significant mathematical knowledge" (p. 216) that can come from classroom interactions.

Sfard (2000) discusses the game of school, in particular, the teaching of mathematics, in terms of meta-discursive rules, or meta-rules. Meta-rules, according to Sfard, are, put simply, the unspoken rules about the sociomathematical norms (McClain & Cobb, 2001; Yackel & Cobb, 1996) and discursive regularities that are present in mathematical classrooms. McClain and Cobb (2001) differentiate between sociomathematical norms and other social norms in that sociomathematical norms are those classroom norms that are specific to mathematics. Yackel and Cobb (1996) offer, as examples of sociomathematical norms, determination of what counts as different, sophisticated, and efficient, and elegant mathematical solutions and explanations. Sfard (2000) argues that the meta-rules which govern classroom discourse are much more implicit than explicit. Lampert (1990) asserts that although meta-discursive rules can be made explicit, students learn the meta-rules by participation in classroom interactions. Meta-rules act as social and sociomathematical norms and "are value laden and count as preferred ways of behavior" (Sfard, 2000, p. 170).

Discursive Devices

Through the use of particular discursive devices, teachers perhaps unwittingly deny student authority over mathematical knowledge and the development of mathematical agency. Some of the devices used frequently and the ones of most interest for this study are revoicing, echoing, cloze-type questions, use of the word *we*, questioning strategies, and dismissive responses.

Revoicing is one such device that teachers use and can help or hinder communication in the classroom. O'Connor and Michaels (1993) describe revoicing as the redecorating of a student's response by the teacher, but with a definite power dynamic. If a teacher revoices a response in such a way that changes the meaning of a student's explanation, then the teacher may unwittingly discourage future communication by removing any authority that students have assumed. If, on the other hand, a teacher works to affirm a student's answer and revoices it simply for the purpose of clarification, the students may still retain their sense of authority. Forman and Ansell (2001) assert that one of the effects of revoicing is that "student explanations are … legitimated by being animated by the teacher who is the powerful, authority figure in the classroom" (p. 119).

Another of the purposes and, perhaps, goals of revoicing is to "create alignments and oppositions in an argument" (Forman, Larreamendy-Joerns, Stein, & Brown, 1998a, p. 531). Because they often decide on the milieu and participants of a mathematical argument, teachers can covertly assert classroom and mathematical authority (Forman et al., 1998a).

Empson (2003) and others (e.g., Cazden, 2001; O'Connors & Michaels, 1993) discuss ways revoicing can be positively used to redirect student perspectives and develop students' mathematical confidence. Empson (2003) found that if teachers are able to extend the students' "partial success" (p. 338), by animation and revoicing, then they could enhance their students' mathematical identities. In order to buy time, emphasize points, or assess understanding, adept teachers might ask students to revoice their own or other students' explanations (O'Connor, 2001). Although Bill, Leer, Reams, and Resnick (1992) acknowledge the cognitive and agentive benefits for students that revoicing can have, they also admit that revoicing can be used as a device of conversational control.

Pimm (1987) describes many sacrificial strategies, or "gambits", that teachers use to exert their authority in classroom discourse. Three of the most prominent of these gambits are echoing, *cloze* questions, and the use of the word *we*. Pimm calls redecorating and/or repeating student responses echoing. Based on the fill-in-the-missing-word cloze procedure of reading assessment, Pimm describes the use of cloze questions, often in conjunction with echoing, critiquing the narrow scope of this type of authoritative discourse. A student's answer to a cloze type question is either right or wrong with little to no margin of error. If a teacher is looking for a specific answer to a question, then, in order to answer the question, students have to play the game of "guess what the teacher is thinking." In addition, this type of question can refuse students the opportunity to expand on their thoughts past a single predetermined answer and, when combined with echoing, can serve to retain all authority.

Another way teachers exert authority in classrooms was with the use of the word *we* (Pimm, 1987). According to Fortanet (2004), "In the negotiation of meaning that is always present between the person issuing a message and the person receiving the message one of the key elements is the reference of the personal pronouns" (p. 46).*We* can be used by teachers in classrooms as a way of conveying "social convention" without any explanation or justification, thus giving teachers the power of suggestion and refusing students' rights to oppose the "way things should be done here" (Pimm, 1987, p.69). Rowland (1999) argues that students of teachers that inappropriately use *we,* where *you* or *I* more accurately represented the intended referent, have no choice but to accept what was being forced on them. Any original voiced opposition to being covered by the umbrella of *we*, Pimm argues, serves as a potential threat to the conventions of the common pedagogic discourse. Pimm (1984) asserts that students learned the inappropriate pronominal usage of *we*, *us*, and *our*, from their teacher as the accepted way the language of mathematics was spoken and will replicate this misuse in mathematics classrooms to the detriment of their own mathematical authority. Rowland (1999) argues that this reproduction is what teachers that prefer *we* to *you* expect.

Pennycook (1994) illuminates the differences between opposing prescriptivist and descriptivist views on language, particularly on pronominal usage. Pennycook anonymously quotes that "there is never an unproblematic 'we'" (p. 174). Pennycook describes much of the politics behind each of the pronouns of modern English. Pennycook concludes that, as a teacher, there is never an unproblematic use of any pronoun and "rather than being neutral referents of an unproblematic world, their use opens up a whole series of questions about language, power, and representation" (p. 178). The pronoun 'we' is particularly problematic as it is simultaneously inclusive and exclusive. We defines those included in the group as well as those not included, i.e., an Other.

Rounds (1987) investigated the use of personal pronouns in a university mathematics classroom. She found that the teacher avoided third person pronouns, such as *they*, and instead, inappropriately redefined second and first person pronouns, using *I*, an exclusive- *we*, or an inclusive *we* to substitute for a more appropriate *mathematicians* or *they*. On one occasion, the teacher uses an inclusive-*we* when discussing who is defining a function as differentiable. Rounds contends that "by rules that delineate the discipline of mathematics, the inclusive-*we* of students and teacher cannot perform the functions of naming and defining on their own recognizance" (p. 18). She continues that "[students] must make use of 'authorized' terminology" (p. 18). Rounds argues that the teacher qualifies to be a member of the naming and defining community and should have used either *they* or an exclusive-*we*, while implying that students were clearly not members of this exclusive group of mathematical experts. Rounds' basis for her analysis of the teacher's use of *we* is particularly problematic in that it denies that mathematical agency resides anywhere except in mathematicians or those deemed worthy of naming and defining mathematical objects. This attitude toward the nature of mathematics and its teaching and learning directly contradict constructivist principles and inhibit student ownership of mathematics.

Rounds (1987) does recognize that teachers can use *we* as both inclusive and exclusive, placing himself/herself at the intersection of two groups: mathematicians, or those mathematically authorized, and students, or those who are not. This dual use of *we* can be effective in linguistically minimizing “obvious and enduring status differences in terms of mathematical knowledge and classroom role” (p. 23). Rounds insightfully notes that “by developing an atmosphere of communality and consensuality through strategic pronoun use, teachers can avoid erecting barriers to learning that are a result of intellectual alienation” (p. 25).

The methods with which teachers question students, for multiple purposes, may have the effect of enhancing students’ mathematical authority in the classroom. Cooper and Simonds (1999) suggest that in order to promote student participation and students’ ownership of mathematical knowledge, teachers should first and foremost, resist talking all the time. They also recommend that teachers share authoritative roles by asking questions such as “‘What do the rest of you think about that?’” or ‘Does anyone have something they’d like to add?’” (Cooper & Simonds, 1999, p. 162). In addition to asking such questions, however, the authors also make the point of providing the proper amount of wait-time after asking the question because it is important for the students to have time enough to fully “digest” the content of the question before answering.

One of the most commonly used questioning frameworks is the IRE sequence, where the teacher initiates the sequence with a question, a student or students respond(s), and then the teacher evaluates the response (Cazden, 2001; Mehan, 1979; Wood, 1992). Edwards and Mercer (1987) found an alternative to the sequence where the teacher’s evaluation of the student’s response was replaced with or added to by an expansion of and on the student’s response. The differences between the types of interactional sequences are profound. The former reduces students’ knowledge and understanding to being able to respond to their teacher's questions (Edwards & Mercer, 1987) and reduces student knowledge to bits of information. Conversely, the latter IRE sequence creates a learning community where knowledge "is constructed by and with those involved in the learning activities, by teachers and children alike" (Buzzelli, 1996, p. 525). The type of IRE sequence used by a teacher is therefore very important and should reflect the teacher's goals and beliefs about teaching and learning. The IRE sequence and its derivatives have been prominent in many studies on mathematics classroom discourse (e.g., Buzzelli, 1996; Nardi & Steward, 2003; Renne, 1996)

Teacher Reflection

Mathematics teachers' may not be aware of the value of educational research or may believe that educational research holds no value to practicing teachers (Senger, 1999). However, Senger does provide evidence of the ways teachers can initiate, at least, some changes in beliefs and practices from a teacher-as-authority approach toward a more constructive, student-centered approach to teaching through reflection and discussion with colleagues. An improvement in consistency between reform efforts in mathematics education and teaching practices via an increase in teacher reflection is the primary goal of this study. Reflection is an oft used term in teaching and teacher education. But what is reflection and how do teachers reflect on their own teaching practices?

Dewey (1933) defined reflection as the "active, persistent and careful consideration of any belief or supposed form of knowledge in the light of the grounds that support it and the further conclusions to which it tends" (p. 9). Dewey also warns that there does not exist a specific set of steps for teachers to follow in order to be reflective. Jay and Johnson (2002) provide a working definition of reflection that provides specific insights that are more useful at a practical level than Dewey’s original definition:

> Reflection is a process, both individual and collaborative, involving experience and uncertainty. It is comprised of identifying questions and key elements of a matter that has emerged as significant, then taking one's thoughts into dialogue with oneself and with others. One evaluates insights gained from that process with reference to: (1) additional perspectives, (2) one's own values, experiences, and beliefs, and (3) the larger context within which the questions are raised. Through reflection, one reaches newfound clarity, on which one bases changes in action or disposition. New questions naturally arise, and the process spirals onward. (p. 76)

Conclusion

The literature reviewed in this chapter illuminates the current and historical views of various and related topics in mathematics education. In particular, I have attempted to show the relationships between these topics of reform, constructivism, authoritative discourse, the various uses of language in the classroom, and reflection.

CHAPTER III

RESEARCH METHODS

Overview

The purpose of the following chapter is to detail the methods and procedures I used in this study. I begin with a brief description of how this research makes use of and is situated within the qualitative research paradigm. This chapter also describes the study participants and the site. This is followed by a detailed description of data collection procedures and the methods employed to analyze these data. Throughout the remainder of the dissertation, I will refer to the teacher at the focus of my research as Ms. M or the teacher.

The Qualitative Paradigm

Why is a qualitative perspective well suited to my research? This study was focused on the relationship between a teacher's intentions and discourse practices. Thus, an interpretive approach was better suited to explore this relationship. In particular, I proposed that for a teacher whose intention is the development of students' mathematical agency, the words the teacher uses in classroom interactions would be revealing of this alignment between intentions and practices.

In what follows, I discuss some of the ways that agency and mathematical agency have been researched in the recent literature. Stryker asks an important question that researchers and practitioners should pay close attention to and attempt to answer in relation to the development of agency: "The proper question is not whether human social behavior is constrained or constructed; it is both. The proper question is under what circumstances will that behavior be more or less heavily constrained, more or less open to creative constructions” (1987, p.93). How then are this and other questions related to agency and mathematical agency best answered?

Questions of amounts of mathematical agency or the correlation between enacted mathematical agency and student outcomes are not those typically posed by researchers. Instead, questions concerning (mathematical) agency are of a nature more likely to be answered through qualitative research methods. Research in the area of mathematical agency should utilize a combination of any of a number of qualitative methods, e.g., discourse analyses (Boaler & Greeno, 2000; Burton, 2002; Gutstein, 2003), passive and participant observations (Gutstein, 2003), interviews (Boaler & Greeno, 2000; Empson, 2003) and focus groups (Boaler & Greeno, 2000), and textual analyses (Gutstein, 2003). Burton (2002) while researching children’s development of discourse, agency, and a sense of authorship, qualitatively analyzed the narratives produced by children during task-based interviews. The qualitative research of Burton can be considered a type of discourse analysis, although which type was not clearly specified by Burton. This lack of specificity with regard to qualitative methodology is not uncommon among mathematics education researchers (e.g., Boaler & Greeno, 2000; Empson, 2003), but a richer description of the methods used should be provided (cf. Simon, 2004).

Gaining Entry

Gaining entry to the school, the site, the teacher, and the students was not particularly difficult for this research. From the first time I went to the school and the first meeting with Ms. M, everyone I came in contact with appeared open and receptive to having me on campus to observe. There are several potential reasons for their collective attitude toward visitors.

The school was the focus of a large technology grant through Apple computers whereby each student received a laptop computer to use and care for the entire school year. Evaluators from the granting institution and the district frequented classrooms to assess the ways and regularity in which the laptops were used in the classrooms and the ways departments and teachers incorporated the computers into their curricula and lesson plans.

Another potential reason for my easy entry into the school was my direct association with the Middle School Mathematics Project (MSMP). During the previous school year, I observed and video recorded one of Ms. M's classes. The period directly following the one I filmed was Ms. M's planning period. I stayed in her classroom and had a conversation with her about various topics including the taped lesson, the department's curriculum, future lessons, and my unique connection to her as a teacher. During the 2002-2003 school year, I taught in the same classroom, in the same high school, and had many of the same students as did she during the previous year. We also discussed possible strategies for which she planned to teach the topic that followed the one I observed.

Before observations were to begin, Mary Margaret Capraro and I went to see Ms. M in her classroom during her planning period. Dr. Capraro called her to preface our visit. We went to ask if Ms. M was willing to let me observe her while she taught for an extended period during the fall and spring 2005 semester. I told her that I wanted to observe only one of her classes and would be there almost every day to observe for a week before I started video taping.

During this first meeting, I did most of the talking about my work and the observations. I told Ms. M that I did not want her to do anything special, like teaching a specified lesson. I told her that I was interested in seeing her just as she was. The only indication of what I was looking for that I gave her was that I was interested in classroom interaction, specifically how she interacted with her students. I conveyed to her that, from viewing previously recorded video of her and from being in her classroom the previous year, I knew she was a teacher that used a very interactive approach to teaching. She came across as very blasé; however she also seemed to hold back that she was a little pleased that I was interested. She asked why I would want to observe her, as though she was not worthy of learning from her as a model teacher. As a partial answer to this facetious question, Dr. Capraro and I were very complimentary of her teaching.

Participants

The research questions for this dissertation were specifically aimed at a target of reform oriented teachers. Therefore, I was interested in finding a teacher whose teaching practices closely followed many of the principles and standards of NCTM (2000). Of the 15 currently participating teachers in the Middle School Mathematics Project at Texas A&M University, one teacher stood out as particularly representative of my qualifications as mentioned previously. The teacher was chosen for the study because of her highly interactive style of teaching as witnessed from extant video data and previous classroom observations and discussions. Additionally, the teacher was a non-traditional, reform-oriented teacher using a variety of teaching methods including instructional technology and collaborative group work.

The Teacher

The teacher has greater than five years experience and is in her third year at her current position. Through interviews, I learned that Ms. M has been teaching mathematics at the middle school level for approximately fourteen years. She spent one year teaching at the high school level before returning to teaching seventh and eighth-grade mathematics. Ms. M described herself as “pretty experienced, curriculum-wise” and having a “pretty good grasp of mathematics.”

The intended textbook for Ms. M’s class was *Math Thematics*, a highly rated, National Science Foundation funded mathematics textbook. Ms. M did not use this text by choice. Although, admittedly, she felt that she was not trained adequately to effectively use this text, she did not see the added benefit of using *Math Thematics* over the curriculum she and other teachers put together. Whether the decision to use an alternative curriculum from the agreed upon textbook was a sound decision based on the needs of the students, by not using the textbook, Ms. M demonstrated some level of teacher agency.

Ms. M has an intern this semester that teaches eight lessons during the semester. Ms. M told us that the intern, whom I will call Laura, would probably not be teaching during the observation period. Laura was a student in a problem solving class I taught at Texas A&M University during the spring semester of 2004. Laura was a very good student, but my relationship with her was and is very superficial. While in my class, she never came to my office for help. The emails between Laura and me were always to relay class information or homework assignments and never for assistance or on a personal level. Laura was usually “all smiles” while in my class. She was enthusiastic about teaching and mathematics and was very social.

The Class

The class that I observed was a Pre-AP eighth-grade mathematics class. There were no differences between the curriculum of the Pre-AP and Ms. M’s regular eighth-grade classes. There was an implied difference in expectations between Pre-AP and regular eighth-grade students, but this difference was also slight and inconsequential., because students were placed in the Pre-AP class at the request of their parents. A parental request was the only requirement for students to enter the Pre-AP program. According to Ms. M, there were “no grade requirements or [test] score requirements to get in.” However, most of the students in Ms. M’s class were appropriately placed with a few exceptions. In an interview, Ms. M told me that there were some students that should be taking eighth-grade algebra instead of their current class and some that were struggling to keep up with the level of mathematics taught in a Pre-AP class, but the majority of the students were placed appropriately for the content level of her class.

The school district in which Ms. M teaches is a central Texas, rural, highly diverse, relatively low socioeconomic status school district. The participating class was a Pre-AP eighth-grade mathematics class of 28 students. There were 11 males and 17 females in the class. There were 14 White students, 2 Black females, and 11 Latino students in the class. The demographics of the participating class were relatively similar to those of the school district and school. Ms. M told me that in order for the class to be more representative of the school, she "would need two more Black students…and two or three more Hispanics." She said that the class was "more heavily White than in the school."

While meeting with Ms. M, I learned of another aspect of the school culture that I did not expect. This year, every student, with a few exceptions, was issued an Apple notebook computer. The students bring their computer wherever they go and take them home at night. Students and their parents attended training on how to use and care for the computers before they were issued them. I'm interested in seeing how the classroom is different because of the computers

The Classroom Setting

Ms. M's classroom was arranged in straight rows, eight by four, with every desk facing the front wall. There were windows behind the students made from frosted glass blocks out of which the students could not see. There was a small chalkboard on the front wall. The walls were adorned with many posters, banners, and other decor conveying various mathematical information such as conversion charts and problem solving tips as well as some with motivational messages.

There is a "Success is a do-it-yourself project" banner displayed prominently on the wall facing the students. The school mission: "Our mission at [the school] is to achieve excellence in a respectful environment where everyone and everything teaches" is on a poster also on the front wall. Beside this poster is another with the school rules: "Cooperate with your teachers and classmates. Respect the rights and property of others.

Follow the student code of conduct. Carry out your basic student responsibilities." Also on the front wall is a Classroom Etiquette poster. On this poster read "1. Say 'Please,' 'Thank you,' and 'I'm sorry.'…4. Pay attention when others are talking. Act interested (Even when you're not)…7. Ask questions when you don't understand….9. Be open to new ideas…." Another poster read "A mind is like a parachute…it works best when it is open." A set of three posters on the front wall read "Think Creatively," "Knowledge is power," and "Try."

On the bulletin board was posted an 8.5 by 11 inch "Ladder of Success" reading "100% I did, 90% I will, 80% I can, 70% I think I can, 60% I might, 50% I think I might, 40% What is it, 30% I wish I could, 20% I don't know, 10% I can't, 0% I won't" Also on the bulletin board, another paper read "A short course in human relations. The six most important words are: I admit I made a mistake. The five most important words are: You did a good job. The four most important words are: What is your opinion. The three most important words are: If you please. The two most important words are: Thank you. The most important word is: We. The least important word is: I."

On a side wall was a banner that read "Have the courage to be yourself." On this wall was also a banner which read "Success is getting up just one more time than you fell down." On the same wall was a poster that read "We support our troops." On the other side wall, two different posters read "Spread your wings and soar to great heights" and "I can because I think I can."

Procedures for Data Collection

To gain an understanding of the daily interactions and the construction of the sociomathematical norms in the teacher's classroom, I observed 20 daily lessons involving one class of students over two months resulting in the videotaping of 10 classes over several weeks. The six most appropriate of the classes were transcribed using simplified Jeffersonian notation (Jefferson, 1984). A description of the transcript notation can be found in Appendix A. The appropriateness of the lesson was judged on the basis of the amount of interaction that occurred or could have occurred. Classroom activities that were not transcribed or videotaped were those where students and the teacher would obviously not be engaged in a meaningful discourse centered on mathematics, e.g., a testing situation. The transcription of one of the lessons is provided in Appendix B to illustrate the ways both the transcript notation was used and a snapshot of the classroom events.

Video/Audio Tapings

As a participant in the MSMP project for three years, Ms. M had been video taped for lessons of the previous two years. Previously, graduate students and/or faculty involved with the MSMP project observed and video-taped teachers under instructions that they were to teach specific and agreed upon lessons covering the topics of data and probability, number, and algebra, depending on the grade level being taught. The difference between project tapings and the ones for this research was that the teacher was not given specific direction as to the lesson or text to be to use on any given day.

Instead, the teacher was asked to do nothing different than what she would normally do and I would simply observe and record her lessons and classroom interactions.

The video recordings were made with a small, consumer grade digital camcorder mounted on a tripod. Attached to the camera was a receiver for a wireless lapel microphone which the teacher wore throughout each taping. Because my focus was the teacher and her discursive moves, the wireless microphone allowed me to clearly hear every word spoken by the teacher for each observed lesson. Also, I chose to maintain the teacher as the focal point of the video camera throughout each lesson.

One of the choices I had to make while taping was how much of the teacher would fill the frame (Hall, 2000). The question was whether to zoom in close enough to see facial expressions or remain at enough of a distance to view other body language. Also, I had to decide what of the teacher's immediate environment to leave out. What could be happening just off screen? For the majority of the filming, I maintained a medium perspective of the teacher, capturing the teacher's full body as well as 10 to 12 students in each frame. When the teacher worked with small groups, I zoomed in close enough to see the faces and upper bodies of the teacher and all group members. For face-to-face interactions with individual students, I zoomed in close enough so that only the two participants of the discussion were viewable in the frame.

Interviews

I interviewed the teacher based on emergent themes from the classroom observations. The field notes and selected video recordings were used as "stimulated recall" to reconstruct classroom procedures and investigate issues related to the research questions (Morine-Dershimer & Tenenberg, 1981). The interviews were unstructured with questions asked of the teacher such as *what were you trying to accomplish here?* or *what did you mean by this?* with accompanying documentation of the classroom referent. These interviews were intended help re-create the contexts of the classroom and serve to inform the analyses.

My approach to the interviews were as a constructionist (Silverman, 2001). The interactions through interviews were, therefore, treated as part of the discursive data. Because teacher-student interactions were the focus of this study, all other interactions involving the teacher were considered secondary. These other interactions were used as explanatory and confirmatory with regard to results of analyses of teacher-student interactions.

Additional Documentary Materials

For the purpose of triangulation (Anfara, Brown, & Mangione, 2002; Creswell, 2003; Sevigny, 1981) I used curricular materials distributed or utilized during the time of observation by the teacher in addition to the results of a discourse analysis of classroom interactions, interviews with the teacher, observation field notes, and member-checking. Curricular materials can give likely insight into how instruction is influenced by the content of the material to be taught (Barr, 1987). For example, curricular materials that cover less familiar or more difficult content may influence the teacher to accept only a given solution or solution method. On the other hand, material that covers more familiar and less difficult content may provide the teacher with an opportunity to accept alternative solutions and solution methods.

Confidentiality

In order to ensure confidentiality, I changed the names of all participants. In one of the interviews with Ms. M, she revealed to me that I could say whatever about her I wanted so long as I left her name out of the dissertation. Several times during the classes I observed, the teacher's name was used by students or in intercom messages. These occasions are apparent on the original video tapes and the converted digital versions of the lessons. However, in all transcriptions of classroom video and interviews, the teacher's name has been changed to Ms. M. Also, to ensure confidentiality, all of the original recordings, including video and audio tapes, and all converted digital forms of the observed classes and interviews are in my secured possession and were not shared with faculty or students not directly associated with MSMP.

Trustworthiness

In order for qualitative research to be believable, researchers must develop a level of trustworthiness (Ely, 1991). The job of the researcher is to convince readers that s/he is credible and has taken great care to increase the probability that the findings of qualitative inquiry will yield results credible and justifiable throughout the inquiry process (Lincoln & Guba, 1985). Additionally, researchers increase the level of credibility by having results "approved by the constructors of the multiple realities being constructed" (p. 296).

A level of trustworthiness was ensured using member checks, a code-recode strategy, and consultations with experts in the field of mathematics education. After initial coding of classroom transcripts and preliminary analysis of the data, I returned to

Ms. M for clarification of many of the instances of classroom interactions. Ms. M was read the portions of the transcript in question and shown the accompanying video to stimulate her to recall her intentions and motivations during those times during the lessons. Informal discussions between Ms. M and me, immediately following my observation of lessons, served the purpose of clarification and justification of my initial interpretations of classroom interactions. These informal discussions were not accompanied by video or transcripts but were aided by my field notes, as prompts to remind Ms. M of the interactions, and her memory of the recent events.

Role of the Researcher

Qualitative inquiry is an interpretive endeavor in which the researcher strives to construct the realities of the world in which s/he is interested (Lincoln & Guba, 1985). The role of the researcher is as a lens through which all data, analysis, results, and conclusions are filtered. As a researcher, I made every attempt to minimize bias. However, the natural biases that are inherent in interpretive studies should be laid out for the research community to view and interpret for themselves. As such, the following is a description of my background, including my development as a student, teacher, and researcher, as it relates to these potential biases.

As an undergraduate in college I was a mathematics major. This distinction also classified me as an expert in elementary or introductory mathematics by many including professors and other students enrolled and/or struggling in elementary or introductory mathematics classes. This status afforded me many opportunities to tutor mathematics students in need of my expertise. These tutoring experiences are the first memories that I can recall where I took the role of teacher.

My recent interest in teaching began while I was a graduate student in a mathematics department laden with authoritative discourse. Authority over knowledge resided in the professor and the texts, period. This imbalanced distribution of authority has been a constant throughout my mathematics educational experience.

I embarked on my practice as a secondary mathematics teacher intent upon bringing a different experience than my own to the students in my classroom. As a first year teacher of “average” and “low-achieving” students, my intentions to share mathematical authority and develop mathematical agency became mute for various reasons and I resorted to the controlling methods of teaching and knowing that I experienced as a student and so resented. I found it difficult to separate my authority over the events in the classroom and my authority as an “expert” in the field of high school mathematics. Teaching from the mathematical pulpit became less demanding and more comfortable. On many occasions, I would “encourage” students to be creative with their solutions by modeling multiple ways to solve problems and encouraging multiple solution methods for the same problem. I hoped by modeling multiple solution methods that they would emulate my behavior when they were working on their own or within their groups. I found, instead, that I was trying to force a way of thinking by forcing certain behaviors by using my classroom authority to exert mathematical authority.

I often made feeble attempts to oblige students to verify the mathematical validity of statements and solutions in class by responding to their oft asked question of "is this right?" with questions such as "what do *you* think?" My turnabout questions usually appeared to frustrate the students rather than inspire them to develop argumentative skills and share in the authority to justify and verify mathematics, as I intended. Students would often remind me that I was the teacher, I should know the answer, and, most importantly, I should tell them. Eventually I did tell them and continued to tell them and withhold the mathematical authority that I now believe could have helped them develop their mathematical abilities and thinking. I believe that some reflection would have shown me that my practices did not reflect my intentions.

About midway through my first year of teaching, I made the decision to leave the classroom and continue as a fulltime doctoral student. My frustration with the inconsistency between my beliefs about the learning of mathematics and my teaching practices steered me toward the line of research in this dissertation. The context of authoritative discourse and its effect on student's mathematical agency is but one of many in which I found my practices not reflecting my beliefs or intentions. As a researcher, my shortcomings as a well-intentioned teacher provide me with a critical lens by which to view other teachers' practices.

Data Analysis

The data were collectively analyzed qualitatively. I analyzed the discourse from all transcribed classroom interactions between Ms. M and her students. Private interactions between Ms. M and me or the intern were considered secondary as the focus of the study was the teacher talk in class. The results of these analyses were combined with other data to help inform and answer the research questions. Below, I discuss the nature of discourse analyses and the ways in which I analyzed the teacher-student interactions.

Discourse Analysis

A discussion of discourse analyses and analyses of discourses should not continue without some mention of *discourses* and *classroom discourse*, how the notions of discourse and a discourse are related, and how they differ. A discourse is a set of all communications that take place within a given community, including verbal and non-verbal communication and written communications, as well as ways of acting, interacting, reading, thinking, and valuing (Gee, 1997). A mathematics classroom can be considered one such community with its own discourse (Sfard, 2000). Certainly, individuals do not belong to only one community of discourse and, subsequently, do not engage in only one discourse, in this sense of Gee. Another view of discourse is one of classroom interaction – the verbal and non-verbal communications that occur in the classroom between teacher(s) and student(s) or student(s) and student(s). Although the latter is the view that many people take of the meaning of classroom discourse (a way of communicating), research on classroom discourse also considers those aspects, e.g., written texts, which may be overlooked as contributing to the discourse (the communications) in and of mathematics classrooms.

There are many different and distinctive types of discourse analyses available to the educational researcher. Some are more available than others. I will concentrate the more in-depth discussion to these types of discourse analyses.

Tannen (1989) asserts that discourse analysis can be considered as the analysis of language beyond sentence level, as the language flows and differentiates discourse analysis from linguistic studies that concentrate on smaller units of language such as words or syllables. According to Tannen, the meanings produced by the relationship of two sentences can be very different than the meanings of the individual sentences when considered separately and out of context. Tannen refers to the context of the sentence as its frame and considers framing important in understanding the meaning of discourse.

Most scholars agree that the term discourse analysis is fairly ambiguous. Stubbs (1983), describes the term discourse analysis to be “(a) concerned with language use beyond the boundaries of a sentence/utterance, (b) concerned with the interrelationships between language and society, and (c) as concerned with the interactive dialogic properties of everyday communication” (p. 1).

According to Brown and Yule (1983), the goal of discourse analysis is "to give an account of how forms of language are used in communication” and, more specifically, to investigate "how addressers construct linguistic messages for addressees and how addressees work on linguistic messages in order to interpret them” (p. ix). Discourse analysis

> on the one hand includes the study of linguistic forms and the regularities of their distribution and, on the other hand, involves a consideration of the general principles of interpretation by which people normally make sense of what they hear and read. (p. x)

Schiffrin (1994) compared and contrasted different varieties of discourse analysis, including pragmatics, conversational analysis, variation analysis, interactional sociolinguistics, speech act theory, and ethnography of communication. Schiffrin argued that although there were distinctions that made each of these approaches different, they all had one overarching common purpose. They all attempted “to answer some of the same questions: How do we organize language into units that are larger than sentences? How do we use language to convey information about the world, ourselves, and our social relationships?” (p. viii).

The significance of the commonalities of all these functionalist, or emergent (Hopper, 1988), approaches to discourse analysis, according to Schiffrin (1994), is that they all regard language as social interaction. Additionally, all these approaches to discourse analyses take a functionalist, in opposition to a formalist, perspective, meaning they make the following two major assumptions: “(a) language has functions that are external to the linguistic system itself; (b) external functions influence the internal organization of the linguistic system” (p.22). It is important to reiterate that the main question we seek to answer through discourse analysis is *what are the ways we use language to communicate information about society and our place in society?* More specifically, for the sake of educational research, *what are the ways teachers and students use language to communicate information about education and their place in the classroom?*

Speech act theory. Speech act theory was formulated by the philosopher John L. Austin and developed by his student, John Searle. There also exist many parallels between speech act theory and Wittgenstein’s notion of language-games and his emphasis on usage. Although speech act theory is not a method, per se, of analysis of discourse, its principles lead to such a method. Speech act theory was considered useful in the study of society and education because it redirected the focus from texts themselves to what we do with or through the texts. Here I mean text in a very broad sense incorporating written and spoken language as well other forms of communication. However, Austin and Searle and other speech act theorists were not interested in an analysis of literature and considered it to ignore their rules of speech.

With regard to discourse analysis, speech act theory classifies messages according to their function in communication as opposed to their meaning. Austin proposes that all utterances can be classified as performatives, statements that represent the doing of something (e.g., “We will now check the answers to last night’s homework”), or constatives, statements that can be judged to be either true of false (e.g., “the answer to number one is 13”). According to speech act theory, during an utterance, the speaker performs the *locutionary act*, which is the actual production of utterance, the *illocutionary act*, which is the use or purpose of the utterance, and thirdly and sometimes, the *perlocutionary act*, which is the reaction to be inspired in the hearer of the utterance.

This simplification of spoken language to these rules of speech act theory is not without those with questions and criticism (e.g., Levinson, 1983; Taylor & Cameron, 1987). Some of these criticisms are focused on the felicity conditions that utterances are required to meet to be considered “speech acts.” The felicity conditions can be thought of as rules of engagement. These conditions are usually met in everyday contexts, but in a classroom, many of the conditions are suspended (Labov & Fanshel, 1977). As an example, three of the felicity conditions for questions are that (1) the speaker does not know the answer to the question, (2) the speaker wants to know the answer to the question, and (3) the speaker thinks that the hearer knows the answer to the question. For many questions asked by a classroom teacher of a student, at least one, and often all, of these conditions is usually not met. Teachers often know the answer to a question to see *if* the student has *an* answer. Insights from speech act theory were used in my analyses of interactions.

Pragmatics. Rooted in the writings of H. P. Grice and also influenced by some of Wittgenstein's (1953) writings, especially *Philosophical Investigations*, pragmatics is a theory of how language is used with emphases on meaning, context, and communication. The first concept of Gricean pragmatics, one of the most widely used forms of pragmatics, is speaker meaning, as defined by intention. Grice partitions meaning into categories of natural and non-natural meaning based on an utterance being without or with intentionality, respectively.

An example of non-natural meaning is one that is dependent of some human intention, such as the statement, "That bell means that you should all be in your seats." An example of natural meaning is a statement that is not dependent on any human intention, such as "My shivering means that I am cold."

The second of the concepts crucial to Grice's pragmatics is that of cooperation. The cooperative principle supposes that certain conventions hold together and make interactions possible. Grice's cooperation principle is comprised of four maxims: *quantity*, *quality*, *relation*, and *manner*. *Quantity* refers to the amount of information contributed in an interaction. *Quality* refers to the truthfulness of the contribution. *Relation* refers to the relevance of the contribution. *Manner* refers to the ambiguity and conciseness of the contribution. Gricean pragmatics rests on the assumption that all parties in an interaction are knowledgeable of and follow these maxims.

Several criticisms of Gricean pragmatics exist and should be shared. Slembrouck (2004) points out that one of the major criticisms of Grice is that he supposes that cooperation between speakers is what holds together social interactions. It is this principle of cooperation that supposes that both or more speakers understand and share the same rules and conventions of conversation and interaction. This view ignores issues with power and authority that occur in many interactions, especially those occurring in the classroom between teachers and students. It also disregards cultural differences between teachers and students which often occur in ethnically and racially diverse classrooms.

Another criticism of Gricean pragmatics is concerned with Grice's requirement that there be some historical mutual understanding between those involved in speaking in order for the intentions of one speaker to be known to the other (Sembrouck, 2004). This requirement is most likely one that is not satisfied in many discursive situations. Grice offers a possibility around this criticism in his notion that utterances should be considered as responses to those directly preceding them and that the historical perspectives and understandings of the speaker and listener are fluid and change as interactions occur. Although pragmatics is not the central type of discourse analysis of this dissertation, there are aspects of Gricean pragmatics that can be used in other discourse analyses such as his concept of speaker meaning and intention.

Conversation analysis. Conversation analysis is the sociological analysis of interaction based on work by Sacks (1992). Conversation analysis grew out of the study of ethnomethodology (Schiffrin, 1994). One of the main needs for a different approach was the problem of the "invisibility" of common sense that occurs through ethnomethodological studies. Have (2000) describes this paradoxical issue in ethnomethodology. Ethnomethodology is concerned primarily with what constitutes common sense and the ways what is considered common sense is produced, supported, and reproduced. By basing study of common sense on methods that make use of common sense, common sense becomes an unexamined resource as well as a topic of examination. Ethnomethodologists have some ways of skirting this issue, but these are not the topic of this discussion.

In conversation analysis, the focus is on the sequence of statements or gestures that make up the interaction, likened to "dancing or a joint musical performance" (Stubbe et al., 2003, p. 354). Stubbe and colleagues argue that conversation analysis "rejects the typical linguistic model of communication as sending and receiving messages" (p. 354), and is instead focused on how interactions are jointly constructed by the interplay between participants. The direction of a particular interaction strongly depends on the next speaker's interpretation of the previous utterance or gesture. From a conversation analyst's perspective, there are multiple ways an utterance or gesture can be interpreted and "it is important to find evidence in the interaction of which of these possible interpretations have been taken by the participants" (p. 354).

Conversation analysis is an emergent theory as compared to speech act theory where the possible functions of speech acts are “known” in advance of the respondent’s utterance. The meanings of utterances and gestures are made and, therefore, must be interpreted in context. In conversation analysis social and contextual factors are not taken as definitive of a participant’s involvement in interaction but rather as a resource to help explain the meanings and interpretations of utterances. Stubbe and colleagues claim a strong conversation analysis position on interactional context, namely that “the interaction *is* [italics added] the context” (2003, p. 354).

Critical discourse analysis. Critical discourse analysis (CDA) is a combination of discourse analysis methods with the aim of explaining the uses and abuses of power in discourse(s). Fairclough describes CDA as an interdisciplinary method of analyzing speech and text, in a very broad sense, where language is viewed “as a form of social practice” (1989, p. 20). The goal of CDA, according to Fairclough is to unveil the nature of discourse practices that have, over time, become accepted as common and natural. Fairclough (1992b) explains CDA as three dimensional: “Discursive events (e.g., interviews, conversations, newspaper articles) are analysed linguistically as texts, as instances of discourse practice, and as instances of social practice. By “discourse practice” I mean the practices of producing, distributing, and consuming texts” (p. 269). Fairclough continues with a suggestion that these dimensions can be overlapped and connected to better explain “how particular sorts of text are connected with particular forms of social practice, and how the connections are mediated by the nature of the discourse practice” (1992b, p. 269).

Although CDA can be conducted drawing from a variety of discourse analytic and socio-analytic traditions, there are general practices that are considered standard and accepted. Fairclough (1992a) provides a detailed description of the methods that should be used to conduct a thorough CDA. Fairclough also warns that critics of variation in CDA not be too quick to judge an analysis as not fitting the guidelines of CDA, but instead value these differences that contribute to the growth of the field of CDA. Seidlhofer (2003) argues that the debate over the rigor and appropriate use of CDA is likely to continue as more researchers use this method of discourse analysis.

Conducting the discourse analysis. What follows is Potter and Wetherell's rough outline of how to perform a discourse analysis. Their description purposely gives no details as to which kind of discourse analysis is most appropriate as that is a question for individual researchers on how best to suit their research needs. This outline is supplemented with details of how I conducted my analyses and applied the results to the overall analyses of all of my data.

Potter and Wetherell (1987) provide a seemingly simple set of stages involved in doing discourse analysis. They caution, however, that these stages are not to be used as a recipe. Also, Potter and Wetherell caution that the sequence of these stages is not necessarily the order that they will be completed. The first of the stages is deciding on research questions and deciding whether discourse analysis is an appropriate method for providing answers. The research questions should be centered on discourse "construction in relation to its function" (p. 161).

The next stage is the sample selection. This step should be carefully taken to provide that your source of data will be appropriate to the research questions. The research questions should be the determining factor of your sample size. Potter and Wetherell (1987) suggest that more data do not necessarily mean a more thorough analysis, e.g., more interviews can translate to more work with little benefit if they add nothing to the analysis.

The third stage is collection of records and documents, including field notes, transcripts, journals, etc. The records referred to here should be distinguished from the type of records gathered from direct interaction with the researcher, e.g., student work from a task-based interview. Potter and Wetherell (1987) suggest that a recording be made of any conversations that are to be analyzed and point out that ethical responsibilities require permission for the use of any documents or records.

The fourth stage is interviewing. There should be some caution exercised in this step because the interviewer is an intrusion into the natural setting. Conversational analysts (cf. Stubbe et al., 2003), instead suggest that only talk that is naturally occurring should be analyzed. There is much debate over how to conduct and interpret interviews (cf. Fontana & Frey, 2001) and much care should be given to the decisions made in regards to interviews.

Potter and Wetherell's (1987) fifth stage is transcription. This applies to whatever data are not in written form, e.g., the interviews, classroom interactions, etc. As transcription is an extremely lengthy process, ranging from the simplest of transcripts to the most complex, Jeffersonian style transcript, much care should be given in answering the question of what exactly the researcher hopes to gain from the transcripts. This varies according to the type of discourse analysis that will be performed.

The sixth stage is coding of the data. Potter and Wetherel (1987) suggest coding as a way to initially group the “unwieldy body of discourse into manageable chunks” (p. 167). This initial coding is not where the discourse analysis takes place. Also, Potter and Wetherell take care to note that this type of coding is very different from coding in a quantitative way with hopes of counting numbers of occurrences as in standard techniques of content analysis. Instead, it is a way to both group and gain some intuition about the data.

Potter and Wetherell’s (1987) seventh stage is the actual analysis of discourse. This stage is the most difficult to explain. Potter and Wetherell liken it to riding a bicycle. There are no set of steps that will guarantee successful analysis. Instead, Potter and Wetherell give some useful advice for discourse analysists. They stress that “read[ing] for gist” (p. 168) is the wrong approach and that the nuances and apparent contradictions are where the focus should be. This approach of intuition should have been taken in the previous stage. The efforts of discourse analysis should come in two phases. As suggested by Potter and Wetherell. The first phase is searching for patterns in the data. The second phase is forming hypotheses about the functions and various effects and supporting these hypotheses with evidence from the data.

Potter and Wetherell’s (1987) eighth stage of discourse analysis is validation. Potter and Wetherell provide four main analytic techniques for accomplishing this stage: “(a) coherence, (b) participants’ orientation, (c) new problems, and (d) fruitfulness” (p.

169). Coherence means that the claims made through the analysis bring the whole of the discourse together coherently, without any special unexplained cases. In order for the analysis to be complete, the special cases should have informed and shaped the results. The claims of analysis must also be consistent with the orientations of the participants. This is not to say that the researcher should necessarily ask the participant of his/her orientations, but should garner them from the context of the data. Potter and Wetherell explain that the emergence of new problems actually acts as a validity check on the analysis. Fruitfulness is “the scope of an analytic scheme to make sense of new kinds of discourse and generate novel explanations” (p. 171). This last criterion is not exclusive to the study of discourse and is widely used for validity in scientific studies.

To reiterate a point made above, Potter and Wetherell (1987) stress that there is no method to discourse analysis and the above set of stages cannot be simply followed with the guarantee of a successful analysis. They suggest, instead, the stages should be used as a “springboard, rather than a template” (p. 175).

Discourse Analysis and Mathematics Education

As research in mathematics education has recently taken a turn towards the social (Lerman, 2000b), an interest in classroom discourse and the discourse of the mathematics classroom, in a broader sense, seems to be especially appropriate. Researchers in mathematics education are not psychologists, sociologists, or anthropologists, for the most part, but they employ methods of analyses that originate in these fields. One very broad category of examples of borrowed analytical tools used by mathematics education researchers is discourse analysis.

Recall that the main purpose of functional discourse analyses, in a classroom setting, are finding answers to the question, *what are the ways teachers and students use language to communicate information about education and their place in the classroom?* Questions of social interactions and how these interactions affect how students and teachers situate themselves in mathematics is at the heart of a current trend in situated learning theory in mathematics education. Some examples of work in the area of discourse studies in mathematics education are Chapman (1997), Cobb, Boufi, McClain, and Whitenack (1997), Empson (2003), Gerofsky (1996, 1999), Herbel-Eisenmann, (2002), Morgan (1996, 1998), Morgan, Evans, and Tsatsaroni, (2002), Pimm (1984, 1987), Rowland (1999a, 1999b), and Sfard (2000, 2001), among many others.

Much of the research on discourse and discursive practices do not use “pure” discourse analysis techniques. Instead they use analyses informed by different strands and varieties of discourse analysis. For example, Morgan, Evans, and Tsatsaroni (2002) draw upon the work of Fairclough (1989) and Halliday (1985) to perform a textual and structural analysis of a mathematics classroom. This reforming, revising, and reusing of discourse analytical techniques is not only common and accepted in the field of mathematics education, but is a sign that the field is adaptive and creative.

There are those that question the adaptation of discourse analytic techniques directly applied to the mathematics classroom. Steinbring, Bartolini Bussi, and Sierpinska (1998) offer the following advice to the mathematics education researchers in light of recent research trends in the application of discourse analysis to mathematics education:

> Mathematics education research is slowly learning from discourse analyses…but…it is increasingly obvious that original methods, adapted for the purposes and interests of this particular domain, must be devised. Discourse analysis in mathematics education must take into account the specific mathematical content of the conversations, and its basic unit of analysis cannot be a word or even an utterance but an episode whose unity is decided by one common mathematical or metamathematical question or problem. Therefore, not looking to discourse analysis as a sole source of inspiration and model, mathematics educators use a wide variety of approaches, analytic tools, and ideas. (p. 343)

The discourse of the six chosen lessons was coded for the teacher's use of several authoritative discursive devices, including the use of cloze-type questions, questions or responses that appear to transfer mathematical authority, echoing, revoicing, and various non-literal uses of personal pronouns, e.g., *we*, *us*, and *our*. Other discursive strategies that emerged, such as the use of humor and sarcasm, were coded for following the initial coding. Each of the coded statements for each of the discursive devices was organized and analyzed according to a combination of discursive strategies described previously with an emphasis on a critical discourse analysis perspective.

I read each of the transcripts twice before coding began; the first time with the accompanying video to insure accuracy, and the second time to reacquaint myself with the interactions and to begin to recognize occurrences of authoritative discourse. Each of the six transcripts was then coded for predetermined categories of authoritative discursive strategies, including the uses of cloze-type questions, echoing, revoicing, uses of the pronouns *we*, *us*, and *our*, and statements that appear to transfer authority to students. For each category, I read the transcripts focusing only on that category. After the initial coding of the transcripts, I began to recognize patterns in the teacher's discursive choices, some of which I considered authoritative in nature. The second round of coding produced several new categories of authoritative discourse that were combined with and compared to the a priori categories. After the second round of coding, patterns in Ms. M's speech began to show the relationships between categories of speech and the ways Ms. M used authoritative discourse with respect to mathematical authority and agency. The next chapter presents the results of the analyses of interactions and other data.

CHAPTER IV

RESULTS

The focus of this research is classroom discourse rather than specific mathematical content of a middle school classroom. Therefore, descriptions and details of mathematical content will be provided in what follows only to the extent that they help to frame a discussion of the teacher's discursive moves and how they assist answering the research questions. I begin the results of my research with descriptions of the structures of the classroom activities that I observed. Included in these descriptions are details about the ways in which the activities were used and were supplemented with discourse between Ms. M and her students. I continue with descriptions of the assumed and emergent structures and themes in Ms. M's classroom discourse practices.

The intention behind conducting the discourse analysis was to develop a research perspective with which I could compare to other data collected, including curricular materials, interviews, and field notes from my observations. Throughout the above results, I have included aspects and influences from each of these supplementary data sources. Although the supplementary data did not influence the discourse analysis, their influences are present in the categorization of both grounded and emergent themes reported in what follows.

Classroom Structure

General structural patterns in Ms. M's classroom activities were only somewhat apparent from the classroom periods I observed. As previously mentioned, Ms. M, as a reform-oriented teacher, took a variety of instructional approaches. This variety was evident in the daily activities of Ms. M's classroom. On some days, the students came in to the classroom to be greeted by a warm-up exercise for which directions were written on the chalkboard. On two of the observed days, students arrived to class to a quiz that was intended to last as long as the warm-ups, approximately 10 minutes. Other times, Ms. M would forgo the introductory activities and continue where the class left off on the previous day. On one occasion, she stated "the reason we're not doing warm-ups is cause I really wanted to have a chance to go through, um, the rest of this packet. Then I want you to do some practice on it…."

While students were involved in their warm-up or quiz, Ms. M and/or the intern would hand back graded papers – previous warm-up or homework assignments or quizzes or tests. As papers were handed back, Ms. M walked throughout the classroom observing students working and would question them regarding present or recent assignments. These questions sometimes were common to many of the students and would lead to a class discussion on the topic. Ms. M conducted her classroom activities in what appeared to be a semi-structured manner where activities of the classroom may be based on the needs of the students or predetermined by Ms. M's schedule.

As is typical of many secondary classrooms, as students become more involved with curricular and extracurricular activities, students are taken out of the classroom for many reasons. During two of the observed days, only about half of the class was present for most of the period. On one of these days, many of the students were being photographed with school related groups such as band or student council. However, on most of the days I observed, almost all of the students were present in the classroom.

The presence of the laptop computers in the classroom added another environment for the opening activities of the class. Warm-ups sometimes took the form of a “gizmo,” a virtual learning environment from http://explorelearning.com. The website defines a gizmo as an “interactive simulation that makes key concepts easier to understand and fun to learn” (ProQuest Information and Learning Company, 2005). Ms. M also occasionally asked students to work on a specified gizmo after completing the day’s assigned work, while other students were finishing their assignment. The computers were also frequently used for homework assignments. Students would email the completed assignment to Ms. M the next day or after they finished the assignment. The computers were also used regularly during daily lessons. On one of the four observed days where collaborative group work was assigned, students completed a web quest (See Appendix C) using their laptop computers. Another lesson taught using the computers involved the use of an online graphing utility Ms. M referred to as Create-a-Graph. On this website, students input data and have a choice of types of graphs to use for their data. The types of graphs include bar, line, area, pie, and XY graphs.

In most of the classes I observed, Ms. M projected a homework assignment, a warm-up, a lesson in the form of a PowerPoint presentation, or an internet-based lesson on the screen at the front of the room. Ms. M frequently had students come to the projector with their computers to present work done on their computers.

Following the warm-up or quiz, Ms. M either questioned students about the previous day’s homework assignment, warm-up, or quiz or prepared students for the current lesson with a review of mathematical topics or concepts to which students had previously been exposed. One interesting pattern in the structure of Ms. M's classroom proceedings was the way she ended every class period. With only one exception in the 20 classes I observed, Ms. M ended class by stating "People please, have a good day. I'll see you tomorrow." The words were almost always uninterrupted, but on the occasion that they were interrupted, they were all but once stated and in this order. Next, I present the findings from my analysis of Ms. M's classroom discourse.

Discursive Devices

The transcripts of the six lessons I observed were initially coded according to predetermined categories of authoritative discursive strategies or moves. These strategies included the uses of cloze-type questions, echoing, revoicing, uses of the pronouns *we*, *us*, and *our*, and statements that appear to transfer authority to students. Below, I discuss the ways each of these strategies was used with examples from the transcripts. After the initial coding of the transcripts, I recognized more categories of speech patterns in the discursive moves of the teacher, some of which I considered instantiations of authoritative discourse. These discursive moves included the use of directives and definitive statements, statements of verification, humor, including sarcasm, statements that appear to allow students "behind the curtain," so to speak, and statements that encouraged students' sense of responsibility and ownership. It also became necessary to differentiate between the meanings, motivations, intentions, and reactions to the utterances of the teacher. After the second round of coding, the coded transcripts began to reveal patterns in Ms. M's speech that showed the relationships between categories of speech and the ways that some of the categories overlapped.

In representing Ms. M's use of authoritative discourse, I discuss these categories and show how the categories emerged through examples from the classroom interactions. I continue with the ways the results from the analysis of classroom discourse combine with the interview data, the observation notes, and the curricular materials to answer the research questions.

With each of the categories of authoritative discourse discussed below, I use excerpts from the transcribed classes. The conventions used in all transcripts, including those of the interviews, are described in Appendix A. If I felt that particular conventions were not informative for the purposes of illustration, the conventions were removed from the excerpts before inclusion in text for the sake of readability.

A discursive strategy shown to be used in an authoritative manner is only useful in analyzing a teacher's use of authoritative discourse to the extent that the teacher uses the device. If the teacher does not regularly make use of a particular strategy, then very little can be gained from analyzing the discourse from the perspective that this unutilized strategy is pertinent to the teacher's teaching practices. In a previously unpublished but related study (Harbaugh, 2004), I found that, through an analysis of a teacher's classroom discourse choices, including cloze-type questions, echoing, and revoicing among others, such strategies were used extensively and in a way that showed the teacher's exertion and conflation of pedagogical and mathematical authority. Using the previous study to inform the current one, I chose to approach Ms. M's classroom discourse from a perspective that initially highlighted these strategies and others.

My first attempt at coding the transcripts of the video recorded classrooms included coding for cloze-type questions, echoing, revoicing, uses of *we*, *us*, and *our*, and statements and questions that transfer mathematical authority. The teacher's statements and questions that transfer mathematical authority to students led to interesting findings discussed later under the category of "encouraging a sense of responsibility and ownership." The teacher's use of the personal pronouns *we*, *us*, and *our* became a primary focus of the analysis and led to further analysis of Ms. M's other personal pronoun usage. In what follows, I discuss the analysis of each of Ms. M's discursive strategies in the areas of cloze-type questions, echoing, and revoicing, uses of the pronouns *we*, *us*, and *our*, the use of directives and definitive statements, statements of verification, humor, including sarcasm, statements that appear to allow students "behind the curtain," so to speak, and statements that encouraged students' sense of responsibility and ownership.

Cloze-Type Questions

The first of the discursive devices discussed here are cloze-type questions. As a refresher of cloze-type questions, I provide the following discussion. In reading assessment, the cloze procedure is where words are omitted from a reading passage and the reader is asked to fill in the blanks to test for reading comprehension. Following Pimm's (1987) discussion of the ways mathematics teachers use a questioning strategy similar to the cloze procedure, I have adapted and extended the category of questions to include all questions that are literally or figuratively intended as fill-in-the-blank questions and called them cloze-type questions. Cloze-type questions were considered a category of questions where the teacher has a preconceived answer in mind. An example of a cloze question is "And this one's going to be _____?" This question would be verbalized by the teacher prolonging the long e of "be" at the end of the question. Another version of this question, and one that I also considered a cloze-type question would be "What would this one be?" There was one and only one acceptable answer to this question and the teacher was only willing to accept the correct answer. I provide several examples of Ms. M's use of cloze-type questions below.

Ms. M used cloze-type questions as a discourse strategy in various ways. Many of these uses were very literally cloze questions. Other questions were classified this way because there was only one specific answer that was accepted as the correct answer and Ms. M re-worded or repeated the question until the correct answer was provided by either a student or by Ms. M. Below I provide several examples of the ways Ms. M used cloze-type questions in the classroom.

In the first example, Ms. M is discussing how changing the scale factor changes the dimensions of a dilated figure. Ms. M poses a hypothetical situation for students to consider using the projected presentation as a visual. The cloze-type questions in the excerpt below are italicized.

Ms. M: ...Okay. It says the distance from the center of the dilation to each point is equal to the center of the dilation to each corresponding point of the original factor times the scale, ur, the original figure times the scale factor. Why did they have to do this to us? Fine. What does this mean? What if the distance from here to here is five? ((*pointing to the figure on the screen*)) *What's my scale factor?*

SS: Five.

Ms. M: No. *My scale factor is?*

SS: two

Ms. M: So what, if the distance from here to here, this is my point of origin. From here to here is five, *what's the distance from here to here gonna be then?*

SS: Ten

Ms. M: Ten

The second cloze-type question was a true cloze question. Ms. M used her intonation and prolongation of the word is to provide students a verbal blank in which to fill. The first and third cloze-type questions above were classified as such because of the limited range of answers that were acceptable by Ms. M. This limit was explicitly given in the first case and assumed by the hearer in the third. Notice that Ms. M, in all three cases, was searching for a correct answer to her questions and, in the first case, did not explore why more than one student gave an answer of "five."

In the next example, Ms. M is attempting to remind the students how to plot ordered pairs on the Cartesian plane by narrowing the scope of the possible answers to her questions. Ms. M, using several different ordered pairs, reiterates the same point that the first number of the pair represents the *x* value and the second is the *y* value. The first of the cloze-type questions is classified as such because of the limited possible answers. This limit is shown by the follow-up cloze question, which takes the focus off of the original point of how to plot points and relocates it in memorization of ordered pair nomenclature. She continues this line of questions until all ordered pairs in view are exausted.

Ms. M: I, okay, first. Let's take a look. ((*moves to the front to talk to the class*)). Here it is. When we're graphing negative one, one. What is this negative one, *what letters are we using?*

SS: (inaudible)

Ms. M: *Well, the first one is what letter?*

SS: (inaudible)

Ms. M: x::, *and the second one is the:?*

S: y

Ms. M: y. *This one is the?*

SS: x

Ms. M: *and this one is the?*

SS: y

Ms. M: *This one is the?*

SS: x

Ms. M: *and this one is the?*

SS: y

Ms. M: *This one is the?*

SS: x

Ms. M: *this one is?* You need to write that down Linda.

Directly following this episode, Ms. M refocused the attention of the class on how to plot points once she felt that they had either learned the nomenclature or had mastered the game of saying “x” and then “y.”

The next example of Ms. M’s use of cloze-type questions shows how Ms. M directed student thinking by asking a sequence of pointed questions that led students through the computational path from unknown to known. In this Ms. M is answering student questions on the previous night’s homework problems. The question that leads to this discussion is “I didn’t understand number five.” Ms. M proceeds by masking an explanation of number five with student participation. Again, the cloze-type questions are italicized in the following excerpt.

Ms. M: ...This is where we’re at. We have a three and a seven and a three changed to four and a half. So I gotta figure out what to do with the seven. *Am I adding, subtracting, multiplying, or dividing, to get from three to four and a half?*

S: Adding

Ms. M: ((*shaking her head*)) What have we been doing? *By the sides::?*

S: Oh.

Ms. M: We’re multiplying. So I need to figure out what do need to do to three. *What do I need to multiply it by to get to four and a half?*

S: A half.

Ms. M: A half? *Is three times a half, four and a half?*

S: No.

Ms. M: No. *What is half of three?*

SS: One and five tenths

Ms. M: One and five tenths. So I need to multiply but I need that one and five tenths but *what do I also need?* (2) The:: three I started with, right? ((*pointing to the three on the screen*)) (2) So I need to multiply by, I need the three I started with. What do I multiply my three by to get the three I started with? (2) *Three times what is three?*

SS: One

Ms. M: So I need to multiply it by one to keep the three and:: I need a half again more. *Yes?* ((*SS nod their heads*))

In this example, Ms. M was unable to make the class understand that in order to dilate the figure, each coordinate in the point $(3,7)$ is multiplied by a scale factor of one and a half. She realizes that series of questions are not sufficient to move the class to a state of understanding. Ms. M shows her subsequent frustration and has the class use an activity designed to teach the same content but in a different manner.

The occurrences of cloze and cloze-type questions throughout each of the lessons I observed were numerous and pervasive. When cloze-type questions were used by Ms. M, students were forced to give up almost all control of the discussion to the teacher. The only way students were able to exert influence on the discussion was by not participating. The example above and the subsequent events are an example of the ways Ms. M struggles to maintain pedagogical and mathematical authority.

The supposed intention of using cloze-type questions was to have students answer in a precisely intended manner. As shown above, cloze-type questions often appeared together in a sequence resembling a Socratic method of the teacher leading the student through a series of pointed questions where one question would immediately follow based on the answer from the last. The importance of a teacher's use of sequences of cloze-type questions is that the teacher maintains control of the discourse. The above examples illustrate the ways Ms. M used this Socratic questioning style to maintain control of the classroom discourse.

Echoing and Revoicing

The next authoritative discursive strategies for which I coded transcripts were echoing and revoicing. Echoing occurs when the teacher repeats, verbatim, a student's response to back to the student or to the class. As Pimm (1987) suggested, echoing often accompanies cloze-type questions and is used for two likely intentions – (1) to act as an amplifier for students that do not speak loud enough for the rest of the class to hear and (2) to control the flow and direction of classroom discourse. Although these two reasons for using echoing are distinct, it is possible that they are both considered by a teacher as the motivation for using echoing in classroom discussions. Teachers can also use echoing as a way to affirm and validate student answers. This reason for echoing also demonstrates the teacher's uses of both classroom and mathematical authority.

Revoicing is similar to echoing in that part or all of a student's utterance is repeated, but a revoiced response does not have to be repeated verbatim as with echoing. The reasons teachers use revoicing can include the same reasons as echoing, but revoicing a response can add elegance to or change the meaning of the student's original response. Ms. M's uses of echoing and revoicing were not overwhelming in comparison to other strategies, but each time she echoed or revoiced students' utterances, Ms. M demonstrated a desire to maintain the status quo of classroom and mathematical authority. Below I provide examples of the ways Ms. M used echoing accompanied by contextual and analytical explanations.

As echoing often accompanied cloze-type questions, which often occurred as a sequence of questions, each of the examples provided here will contain more than one instance of echoing although not every example of echoing will be associated with cloze-type questions. The first example shows Ms. M questioning a student about dilating figures by various scale factors. Preceding this discussion is a question about the possibility of having a negative scale factor.

Ms. M: …I think I could multiply it by one and that I could reflect it or flip it or turn it (1) move it, but, I, no. Zero. If I multiply it by zero, it's gone. Okay, if I want it to stay the same, it's one. If I make it, if I multiply it by a number bigger than one, what happens to it? Does it, does my im-, is my, is my, is it bigger or smaller than what I started with?

James: It's bigger

Ms. M: Anything bigger than one, is gonna be an enlargement. Anything from zero to one, not including zero and not including one, is gonna be smaller. If I multiply it by one, what happens?

S: It stays the same

Ms. M: It stays the same, if I multiply it by zero, what happens?

S: It poofs (inaudible)

Ms. M: It poofs, disappears. That's right. There you go. Thank you. Let's see.

Notice that Ms. M first describes to the class what happens to figures for various scale factors and then proceeds to ask students to repeat some of the information back to her. In this case, echoing served to confirm that the students could recall what they had just been told. Echoing also served here as amplification for the students responses. In these cases, the student, denoted by *S* in the excerpt, responded in such a manner that many of the rest of the class could not hear the answers. By echoing the answers "it stays the same" and "it poofs," the rest of the class was included in the conversation.

Notice, in the excerpt, how Ms. M strengthens James' response of "it's bigger" to become a slightly more elegant generalization that "anything [scale factor] bigger than one, is gonna be an enlargement." Ms. M uses the revoiced response as a way to explain contractions, where the scale factor is exclusively between zero and one. The word "poof" was originally used by Ms. M earlier in the transcript, however, she chose to echo and then revoice the student's response of "it poofs" in a slightly more sophisticated way.

Both echoing and revoicing were discussed in an interview with Ms. M. She reported that the reason she both echoes and revoices is to make a linguistic link or bridge to the students. She stated

> I think that I should use the correct vocabulary. I try when we're talking about where to put something. Are we gonna put it in the numerator or the denominator. A lot of times it's easy to slip into 'what's gonna go in the top' and not use 'numerator.' Some of the kids are still lost at that point, so I do try to use those terms interchangeably, maybe 70-30. Try to weigh more heavily on 'numerator' so that by the end of the year, you know, by the end of a long period of time being stuck with me reiterating that, they can think of those….I try to use the correct term, but when it's

> confusing to them, I'll just use them interchangeably until we can get to a point where it's the numerator and they know that.

Echoing and revoicing are used as a literal link between the sometimes unsophisticated language of the student and the formal language of the mathematics community or the language of the teacher, which is not always formal. But, as the teacher is an established member of the mathematics community, her language is "good enough."

In the next example, the class is discussing setting up proportions and solving for missing side lengths of similar figures. The class has moved up in difficulty of problems from using a set of two separate similar figures to what Ms. M called nested figures. The more complex case where two similar figures are drawn so that the smaller figure is drawn using the part or all of the sides of the larger figure is the case Ms. M called nested figures. In this excerpt, there are multiple examples of echoing and revoicing accompanying several cloze-type questions.

Ms. M: The x comes from the::?

S: Big

Ms. M: Big rectangle, so what has to go in the numerator over here?

SS: Forty

Ms. M: The forty, the big: rectangle. It doesn't matter which way you do it, but you gotta do it correctly. Either the big rectangle is up and down, or, in this case, it's across. So the two would go across from the ten. Now, sometimes: what happens is people start looking at these and they like it better when the x is on the other side, (1) It is true that these are equal, it doesn't matter which one's on either side, because I might be able to see from here, but I can definitely, if I put 40 over ten on the left and x over two, (2) then I can look from ten to two and say that I'm going to do what? Woops, hello.

S: Divide by five

Ms. M: Divide by five, and if I divide by five across here, what am I gonna do?

SS: Divide by five

Ms. M: So x should equal?

SS: Eight

Ms. M: Eight. Most of us have a hard time doing it when the x isn't on the right hand side, so is it true that I can put the two over, as long as I leave them the same, like that fraction stays the same. The two over ten, and just move the x and 40 over here:. Can I do that?

SS: Yes.

Ms. M: Yeah…

Ms. M's response to the first student in this excerpt is an example of revoicing that extends the student's answer to make it more complete and match what the complete thought would have been had Ms. M completed her own sentence. By revoicing the answer, Ms. M is showing that she has the mathematical authority to authenticate student answers. One could argue that she wouldn't have asked this type of question unless she had and was willing to exert her authority by judging any answer given.

The next response is revoiced to make the connection between the side length and its origin. The revoiced response is followed by an interesting statement that relates to mathematical authority. Ms. M, in one short utterance, hands over mathematical authority and then takes it right back. Who is the authority that will determine whether or not the correctness of the way students have chosen to "do it?"

The next response, "divide by five," is echoed and is used by Ms. M to lead directly to the next question, whose answer happens to be identically "divide by five." By leading into this question, Ms. M showed that she has an agenda and uses sequenced questions to carry out the agenda. The next echoed response of "eight" is followed by Ms. M revoicing in a slightly different way.

Another way to describe revoicing is figuratively putting words, which students did not speak, in their mouths. This way of thinking about revoicing is exemplified by Ms. M telling students that "most of [them] have a hard time doing it when the x isn't on the right hand side." Through her choice to revoice in this way, Ms. M made explicit her authority to know what students think and where students have a difficult time. In an interview, Ms. M spoke of her abilities to anticipate students' problems as being due primarily to her experience. Of the benefits of her experience, Ms. M stated

> my experience is probably in the sense of I see where kids make mistakes and I can anticipate that and have seen that. So in terms of curriculum, I'm probably better at turning the questions around or being more aware of where they're getting stuck and what the questions are and [am] more comfortable with that.

This less literal form of revoicing is very prevalent throughout much of the discourse that I observed. I provide examples below that illustrate how Ms. M represented the thoughts, knowledge, wants, and abilities of the students in place of their own presentation.

- You're not going to be able to figure out without writing them down.

- We know that seventy’s not passing.
- You would know at this time if you’re being invited for tutorials.
- We know the angles are the same and we know the side lengths are different.
- We know that sometimes we can go across.
- We know that always, cross-multiplication will work.
- You're going to be able to do it.
- We’re confused and we need to see this in a different way.
- We wanna look at this on graph paper, right now.
- What we wanna do is look at….
- You wanna make sure that your other answers are correct however.
- You’ll want to copy….

There existed, throughout the lessons, a correspondence between the instantiations of echoing and revoicing. They almost always occurred while the teacher was speaking for the benefit of the class. This falls directly in line with the numerous occurrences of echoing and revoicing when the lesson was taught as a whole class discussion. This correspondence between occurrences of echoing and revoicing and the small group or whole class nature of the lesson infers that the primary function of echoing and revoicing is amplification; so that the rest of the class may hear the responses to teacher questions.

Ms. M used revoicing often in the same way she used echoing; to repeat statements for the sake of keeping the class synchronized mathematically. However, Ms. M’s use of revoicing was a more elegant form of echoing in that it repeated a student’s statement in words that reflected her own mathematical authority. Also, by expanding the definition of revoicing beyond restating of a student response or question, we see that Ms. M's uses of revoicing showed that she was willing to exert her classroom and mathematical authority over students in her class through her discursive choices.

In each of the examples above and throughout the observed classes where Ms. M used cloze-type questions, echoing, and revoicing, Ms. M controlled the flow of the interaction, continuing or ending the course of the interaction by taking over the idea by controlling the mathematical ides that are allowed to be voiced. By controlling the interactions with students using these devices, Ms. M did little to share authority or confer mathematical agency.

We, Us, and Our

The ambiguous nature of the use of plural personal pronouns is not exclusive to the English language, but, as English is spoken as a first or second language in most of the "West," particularly in the classroom I observed, I will not discuss the grammars of non-English languages. Vassileva (1998) examined the various uses of *I* and *we* in a large corpus of academic writings from each of five different Western languages including English, French, German, Russian, and Bulgarian. Vassileva found four explanations of 'we': "(1) the 'royal we' (*pluralis majestiae*), (2) the 'humble we' (*pluralis modestiae*), (3) the 'authorial we' (*pluralis auctoris*), and (4) the 'collective we' (*pluralis communis*)" (p.173). Quirk et al. (1985) provide a more thorough set of eight categories for the uses of *we* in spoken and written English as (1) generic, (2) inclusive authorial, found in text, e.g., "this leads to the following conclusions…," (3) editorial, found in text to avoid and replace *I*, (4) rhetorical, a special case of the generic *we*, (5) referring to the hearer, e.g., the doctor/patient ‘how are we feeling today?’, (6) referring to a third person, e.g., sarcastically, ‘I guess we woke up on the wrong side of the bed,’ (7) the royal *we*, and (8) nonstandard.

We is not normally considered the plural of *I*, although *I and I,* in Rastafarian slang, is used to mean *we* (Wales, 1996). *Pluralis majestiae* or the *royal we* could also be interpreted literally to mean God and I if one were so inclined. The *royal we* is often illustrated by Queen Victoria’s alleged utterances ‘We are not amused,’ regarding a story her then groom-in-waiting repeated at the queen’s request. Another oft cited illustration of the *royal we* is ‘We are not interested in the possibilities of defeat’ speaking on the Boer War of 1899 (Wales, 1996). Wales (1996) asserts that the *royal we* has not gone the way of powerful monarchs but has, instead, undergone a change in name to the *presidential we* or *premier we* as illustrated by the rhetoric of presidents and prime ministers such as Margaret Thatcher.

The categories listed above are not static and classification of uses of *we* depend heavily on the intentions of the speaker. Often, these intentions are difficult to determine, but through speech patterns, one can come to narrowed conclusions and make likely classifications of *we*. In this section, I will provide multiple and likely explanations anchored in the speech patterns and discursive choices of Ms. M during the observed classes and interviews.

During each of the observed classes, Ms. M used personal pronouns in multiple ways and with various intentions. The focus of my analysis was Ms. M’s use of *we, us,* and *our* in classroom interactions and the ways pronouns were used to convey pedagogical and mathematical authority. In many cases, plural, first-person personal pronouns could have been reasonably substituted with *I*, *you*, *they*, or others.

Initial considerations of the ways Ms. M used singular and plural, first and second pronouns led to some confusion as to her intentions with the use of pronouns. Initially, it appeared as though these pronouns were being used randomly and interchangeably. After further considerations, including a more thorough analysis of the patterns and frequency of pronoun usage in classroom interactions and interviewing Ms. M, directly asking about her uses of pronouns, I came to different conclusions. Ms. M did not consider personal pronouns interchangeable and her intentions in their uses were purposeful. However, there was evidence that some element of randomness did exist in Ms. M's "choice" of personal pronouns. In particular, Ms. M reported that her choice to use *we*, *us*, and *our* was purposeful and was to convey a sense that "we are in this learning of mathematics together."

In this section, I provide many examples to illustrate the ways Ms. M used personal pronouns. The examples provided below do not constitute the entirety of my analysis of Ms. M's discursive practices but instead exemplify one aspect of the overall analysis. I begin with several examples showing each of Ms. M's different referents of *we* and possible or appropriate substitutions for *we*. Accompanying each excerpt is an explanation of how Ms. M used pronouns to situate herself and her students as members of different groups.

The first example concerns pedagogical issues and does not directly relate to mathematical learning. The context of the first example is student grades. The student in this example is concerned about her grade and would like to increase her grade that will soon be reported by Ms. M by turning in a missing assignment before grades are submitted.

S: Can I make it higher if I--

Ms. M: --((*shaking her head*)) *My* grades are due today. *My* grades are due today.--

S: --cause I forgot to turn in the paper that I--

Ms. M: You know what *you* can do is, *you* could put, *you* could put a little note on it and *you* could put it in the box. *I* make no promises. *We* still have stuff that *we*'re grading. It's, *we*'re gonna pick 'em up ((*turn the grades in*)) today at 4:30. So if *I* can get it between now and 4:30, *I* will try, but it will be the very last thing. I mean that's the, I mean, I mean honestly. Yeah.

The *we* here refers to all teachers at the school. Although she refers to the grades to be turned in as hers, by using *we*, Ms. M situates herself as a member of a group that has certain requirements. These requirements constitute the authority by which she must abide regarding acceptance of the student's late assignment. Notice that Ms. M hedges her position by stating that she can "make no promises." The phrase "I mean honestly" carries an undertone of surprise and sarcasm, as if to say "you are honestly waiting until the last possible minute to turn in an assignment that should have been turned in on time and long ago."

Ms. M’s use of *we* here was inclusive. By referring to the group of teachers as *we*, Ms. M excludes the students in the classroom. Ms. M could have reasonably substituted *I* in place of *we* with little consequence to the meaning of the statement. The message, however, would have changed. The persuasive power of Ms. M’s involvement with a group acting under certain restrictions is more than if she were acting alone. The grade submission deadline was imposed on the group in which she includes herself. Who is she to act against the group? This theme of persuasive peer pressure was evident throughout the observed discourse and will be revisited several times throughout this discussion.

The next example begins with a student reading from a lesson on the computer. Ms. M chose to have various students read the introductory material of the lesson. After each student would read a section, Ms. M would provide commentary and lead a short discussion about the previous section of material before continuing with the next student reading. In this excerpt, Brian has just finished reading a section with a literal reading of the prime symbol, calling it “apostrophe.” Ms. M begins her comments with a correction of the pronunciation of the symbols.

Ms. M: Okay, it wouldn’t be read that way, would it? It has that prime symbol. *We*’re gonna call it A prime, B prime, C prime, and D prime. Every time *you* move a point, *you* give it a prime. So when it says A’, B’, C’, D’, how many points did *we* move?

SS: Four.

Ms. M: All four of them, right? If *we*'d only moved one, if *we* had just moved A, it would just be A prime, B, C, D. *We*'d call it the same name. So a prime after each one indicates that *we* moved them one time.

In this excerpt, *we* can be interpreted to represent either all of the mathematics community or all people in the classroom. In either case, this is an inclusive use of *we*. Ms. M is including the students in the group by sharing the conventions of the community of mathematics learners or those that do mathematics.

A source of confusion is Ms. M's use *you* in the statement "Every time *you* move a point, *you* give it a prime." It would have been reasonable for Ms. M instead to have said "Every time *we* move a point, *we* give it a prime." Because she did not, then one could assume that either this was a conscious and purposeful discursive choice or it was not. If the choice was made with purpose, that purpose was likely to emphasize that students would soon be confronted with moving points through transformations of figures and would need to know what they were to call the moved points. This move would be made to forcefully personalize the mathematical conventions that should be followed. This statement also falls under the analytic category of directives. This category is explained in an upcoming section.

If the change from plural to singular was not with purpose, then the notion of Ms. M's random and synonymous use of pronouns is supported. I explain later how this notion is both supported and refuted in different ways by the transcripts and interviews. The context of the next excerpt is an explanation and example of the transformation dilation based on an assignment on the computer. Ms. M begins by reminding students how multiplication and division are similarly used to dilate a figure based on the scale factor.

Ms. M: …Dividing by two is the same thing as multiplying by a half. So *we* would take the four. *We* would multiply it by a half or divide by two, same thing, and get what for *our* new image? I mean *our* new length?

S: two

Ms. M: two units. Okay. It says ‘the distance from the center of the dilation to each point is equal to the center of the dilation to each corresponding point of the original factor times the scale, ur, the original figure times the scale factor.’ Why did *they* have to do this to *us*? Fine. What does this mean? What if the distance from here to here is five? ((*pointing to the figure on the screen*)) What’s *my* scale factor?

SS: Five.

Ms. M: No. *My* scale factor is?

SS: two

Ms. M: So what, if the distance from here to here, this is *my* point of origin. From here to here is five, what’s the distance from here to here gonna be then?

SS: Ten

Ms. M: Ten, because *I*’m gonna take this distance and do what to it?

SS: Multiply it by two

Ms. M: Multiply it by the scale factor.

Notice how this excerpt begins with examples of Ms. M's inclusive use of *we* similar to the last excerpt but goes on to show an abandonment of the group possession of the problem. In the beginning of this selection, Ms. M refers to the dilated image, the length of the image, and the process of dilation as things that belong to or are performed by the group. She refers to *our* image, *our* length, and multiplying by half that *we* would do. In the next thought, her question to the class is concerning "*my* scale factor," an important part of what was *our* process of dilation. Immediately preceding this possessive reference, Ms. M reads from the text, which denotes "*the* scale factor" without any orientation of ownership. By then taking ownership of the scale factor, the process of dilation may become more meaningful, but for whom? What was, just seconds before, a group process has now become an individual process for the teacher as the only active participant. The students have become mathematical bystanders only allowed to fill in the blanks, e.g., "*My* scale factor is?"

The following excerpt reiterates the claim that Ms. M was aware of the pronouns she used and that these pronouns were intentionally chosen. In this example, Randall frames the discussion with a question about the logistics of turning in an assignment completed on the computer and a suggestion as to how the assignment could be turned in. Notice the way Ms. M corrects herself in her third appearance.

Randall: So these are gonna be (inaudible), so *we* don't have to turn them in?

Ms. M: Um. At, I don't think. See I can't remember now if at the bottom of it gives *you* a little thing *you* can take a picture of and email *me* or not.

Randall: Okay.

Ms. M: So *let's* see. Yes *I* would like to do that, but *let's* see what it does. It seems to *me* that it just goes through—

S: (inaudible)

Ms. M: yeah. *I* don't think at the very end it gives it to *you*, cause once *we* wanna do from here, once *we* know what a dilation is, *we* wanna graph it and put it on graph paper and start looking at doing those and *we* have that, *I* have that on paper.

S: So when *we*'re done, do *we* just save (inaudible)?

Randall: I'd take a picture of it just in case.

Randall and the other student show, by their use of *we*, the solidarity of the group of students of which membership does not extend to include their teacher, in this case. Ms. M is not completing the assignment to turn in to herself and does not belong to the group referred to by her use of *we*.

In the next excerpt, Ms. M is attempting to continue the previous day's discussion on the Pythagorean theorem. Ms. M is standing at the front of the class both trying to start the class discussion and get the projector to display the PowerPoint presentation that the class did not finish discussing on the day before. As her thoughts are divided by both goals, she moves discursively back and forth. In what follows, notice Ms. M's different uses of *we*. In particular, notice the ways she positions herself with the class as a group with certain explicit responsibilities for mechanical classroom issues and talking about the mathematical content.

Ms. M: Okay. So, is everybody with *us*? And you know what? I was paying attention, but I promise. I forget exactly where *we* were, didn't, so *let's* do a quick review of what *we*'ve gone through. If *you* can, if *you*'re on the page, that's excellent, um, then *we* can look at these. That way if any, if someone was absent or doesn't remember or whatever, whatever, um, *we* can pick this up ((*continue the discussion*)). How come *we*'re not working ((*referring to the projector*))? Because *we* don't have the power plugged in. Power is a good thing. Electricity. Alright, now. This might work a little bit better. Here *we* go. What happened? ((*addressing a student comment*)) Okay, so what are *we* talking about here? What shapes are *we* talking about?

S: Right triangles.

Ms. M: Just triangles?

S: No, right triangles--

Ms. M: --No. Does this work with anything else?

SS: No.

Ms. M: No. The only thing it works with is right triangles. That is all. That is it. So all *we* want, *we* want to know that it works with right triangles. Oh, this is a connection issue, isn't it? Yeah, yeah, yeah. Hold on, hold on. let me see if I can. Looks like *we*'re just a smidgen blurry. What do *you* think? There *we* go. That's right. Purple, yeah. It's kind of a faded purple. Okay, so right triangles. Only right triangles. The introduction, *we* want to look at, they have a special what?

SS: 90 degree angle.

Ms. M: 90 degree right angle. Kind of just a right angle. Some examples, *we* looked at some. *We*'re gonna see 'em turned and rotated. So *we* need to make sure *we* can recognize the hypotenuse. The, that's the key to everything. That hypotenuse, right? The other two sides. Ayh, they're important, they're the legs, but the hypotenuse is the big noise. That is the one. It has to be by itself. The other two *we*'ll square and add.

Ms. M positions herself with the class in the endeavor of continuing the previous day's discussion by stating "I forget exactly where *we* were…so *let's* do a quick review of what *we*'ve gone through." She continues by prescribing some membership requirements of the class by stating "If *you* can, if *you*'re on the page…then *we* can look at these. That way…*we* can pick this up ((*continue the discussion*))."

In the next statement, "How come *we*'re not working ((*referring to the projector*))?," Ms. M switches ideas to a question, likely to herself, in which she positions herself with the projector. This use of we would classify as nonstandard by the classification of Quirk et al. (1985). This use of *we* is one that stands out as perplexing. One explanation for her choice of we is that the use of *we* was not a conscious decision and was, instead, a continuation of the conventions she previously followed. Another explanation is that Ms. M shares responsibility for projection – a joint effort of herself and the projector. This latter explanation is consistent with Ms. M's other uses of we and her disclosure to me that "*we* are all in this together." Positioning herself with the projector was worth mention as continuing of one of Ms. M's patterns of discursive choices.

Ms. M continues to use *we* by asserting that "*we* want to know that it works with right triangles," and what "*we* want to look at" are right triangles. This persuasive usage of *we* continued the positioning of herself jointly with the students and also asserting the direction and interest of the group. She put herself in the position to make explicit the interest of the class. Alternatively, Ms. M could have stated "*I* want to look at" or "*you* want to look at," but neither of these substitutions would have likely promoted the intended cooperative atmosphere.

The teacher/student and doctor/patient relationships share numerous characteristics. Skelton, Wearn, and Hobbs (2002) studied the uses of *I*, *me*, *we*, and *us* in doctor patient consultations and drew conclusions that show just how similar these pairings are with those of teachers and students. Skelton et al. found in doctor-patient consultations that doctors and patients and their companions together used *I* and *we* in almost exactly opposite frequencies in 70:30 and 30:70 ratios, respectively. These researchers found that doctors used *we* in three distinct ways, two inclusively and one exclusively. The first way was inclusive, used to include the patient and to substitute for "you and I. The second inclusive way doctors used we was to mean all people, as in "we are all subject to the aging process" (p. 487). The third way doctors used we was to refer to we as doctors or as the medical profession.

Doctors exerted their authority to know and used *I* primarily in "thinking" ways and *we* in doing ways, i.e., *I* will decide the best course of action that *we* should take. On the other hand, when patients used we, “they never meant ‘you and I, together, doctor’” (p. 487) and instead always meant the patient and his/her family or companion. Rather than participants in the process, patients considered doctors to be “conduits or coordinators of care” (p. 488). Skelton et al. concluded that the D/P interaction prototypically took the form of “P: I suffer, D: I think, We will act” (p. 487).

Many of the teacher/student, T/S, interactions where Ms. M used *we* can be described using a similar description. The parallel prototypical T/S interaction would look like S: I suffer (mathematically), T: I think, We will act. When the students in Ms. M’s class used *we*, it was an easy conclusion to make that they meant “the class and I” and never meant to include the teacher as a referent of *we*. However, when Ms. M used *we,* she usually meant the “students and I.”

Who Are They?

In several instances throughout each of the observed lessons, Ms. M referred to an unnamed group of people as “they.” As Zupnik (1994) explains, “… first person plural deictic pronouns may fulfill a powerful persuasive function since they have the potential to encode group memberships and identifications: speakers may index different groups as included in the scope of the pronoun ‘we’ while excluding others” (p. 340). Several examples of such instances are provided below. Each instance is provided out of context and will be accompanied with an explanation of the context of the reference.

Ms. M: Pupils. When you use a, when you, like on those old sailing ships, *they* had, what is that thing that *they* pull out ((*telescoping motion with her hands*)) What's that thing called? It's called a, um,

S: telescope

In these first references to *they*, it should be clear that Ms. M is referring to people of the past that sailed ships. This first reference is non-mathematical and serves only as an example of how the pronoun *they* can be used to incite images of people likely unknown to the students. In this instance, a more specific substitution of *they* with *sailors* or *pirates* could have been made, but the choice of *they* probably had no bearing on the mathematical content or the ability of students to understand the reference. In the next example, the use of they is of a more subtly suggestive nature.

Ms. M: …We would multiply it by a half or divide by two, same thing, and get what for our new image? I mean our new length?

S: Two

Ms. M: Two units. Okay. It says the distance from the center of the dilation to each point is equal to the center of the dilation to each corresponding point of the original factor times the scale, ur, the original figure times the scale factor. Why did *they* have to do this to us? Fine. What does this mean? What if the distance from here to here is five? ((*pointing to the figure on the screen*)) What's my scale factor?

In this situation, Ms. M was teaching a lesson on dilations working with a PowerPoint presentation for which she was not the author. The *they* in this case was the author(s) of the PowerPoint presentation. The author(s) may or may not have been known to the teacher, but the students were never told who the author(s) were and were not aware of the mathematical status of the author(s). They may have been mathematicians, teachers, or students for all the students in this class knew. The teacher made a strategic choice to position herself inclusively with the students by excluding herself from the group of people that did “this to us [the class].”

For the next group of episodes, *they* takes on a slightly different meaning with different suggestions. In the next example, the class is coming to the end of a discussion surrounding similar figures and how similar figures share corresponding congruent angles.

Jeffrey: How would you figure it out if *we* just had the 90 degree angle?

Ms. M: *You* wouldn’t. *You* wouldn’t be able to tell. *They* would have, for a triangle, *they* would have to give *you* one other angle.

Here, Jeffrey was asking Ms. M what she would do in the situation where just the right angle was identified. A literal translation of Jeffrey’s uses of *you* and *we* shows that Jeffrey considers Ms. M able to speak and act mathematically for the group of individuals in the predicament. Ms. M counters by telling Jeffrey what *he* would or wouldn’t be able to do rather than answering the question in the first person. This could be interpreted as an attempt by Ms. M to transfer mathematical authority to Jeffrey. Also notice that *they* could be understood to denote several different, but not necessarily distinct, groups of people. These groups could include mathematicians, other teachers, authors of the assignment, or authors of state mathematics assessments, specifically the TAKS.

The next example shows Ms. M's often repeated instructions on the way to deal with a question about rounding. Several times, while I observed the class, rounding was a source of confusion for the students. Also, Ms. M stated to the class that rounding was frequently a problem and the ways to deal with rounding. The following is one such account.

Bernice: Do *we* round? Or do *we* just leave it?

Ms. M: Just leave it. The only time *you* would round is if *they* ask *you* to round.

Notice first that, in this instance, Ms. M positioned herself apart from the class but not with the authors of the assignment either. Ms. M did not include herself as a likely authority or author that would be giving directions and providing given conditions. If by not continuing the use of Bernice's *we*, Ms. M did not include herself in the group of people that have to strictly follow the directions of the assignment. Ms. M could round if she wanted to according to this statement. *They* is used in a similar way to the previous example. *They* can be taken to most literally mean authors of assignments, including texts, tests and assignments.

The context of the next excerpt was also rounding. The excerpt comes from a different time period than the preceding example, but the players and the content are similar. In this example, Ms. M provides Bernice with a more lengthy explanation of her thoughts and position on rounding.

Ms. M: …Why *you* guys rounded? How come *you* just do this rounding thing?

Bernice: I didn't.

Ms. M: Really? Cause this is five and 625 ten thousandths and this is five and six hundredths? *I* just think that's rounded. *I* mean, *I* don't know, Bernice.

Bernice: We not supposed to round?

Ms. M: No, not unless *they* give *you* permission, unless *they* tell *you* to round…. *I* really wanna know why *you* guys round. If *they* tell *you*. round to the nearest tenth, then *you* have official permission to round to the nearest tenth. If *they* tell *you* to round to the nearest hundredth, *you* have official permission to round to the nearest hundredth. If *they* do not, *you* don't. Okay?

In this example, Ms. M begins suggestively pleading with the class, in particular, Bernice, to not round their answers. Once Bernice has been convinced that she did indeed round, she changes her stance to claiming ignorance of a rounding convention. Ms. M responds by explaining the convention of rounding that Bernice and the rest of the class should follow. The message in this example is that students must depend on directions and, ultimately, some higher mathematical authority to make decisions on their mathematical actions.

An appropriate alternative explanation of a convention on rounding which explained various reasons for rounding and under what circumstances rounding was most and least appropriate could have been provided to the students. With this understanding, students could then make decisions about whether or not to round their answers. Instead of such an explanation, Ms. M chose to defer to other mathematical authorities when, really the decision whether one should round, for this assignment, was hers.

The next example is very similar to the last, but with a profound difference in the teacher's positioning of herself thorough her pronoun choices. This interaction occurs at a point after Ms. M was explaining the meaning of quadrilateral to the class. She continued sequentially naming figures with increasing numbers of sides until she was halted by her lapse in memory of the term hendecagon.

S: I didn't know there was an eleven sided figure.

Ms. M: Sure, there's an eleven sided figure. I just, I would call it, I don't know off the top of my head what its name is. Alright, here *we* go. *We* have this. Now, this is the deal. Look on your paper and *they*'re telling *us* something. This is exactly like what *we* were looking at on the graph yesterday. *We* just didn't have coordinates for it. *We're* going to change it to a scale factor of two.

In this instance, *they* are telling *us* something rather than, as before, *they* telling *you* and not *me* (the teacher). To be clear, the teacher has situated herself within the group of learners/students/subjects to be instructed and informed of something important by members of a distinctly different group.

Another interesting aspect of this excerpt is a hinting at or glimpse of teacher agency. When Ms. M says that she doesn't know the name of an eleven sided polygon but that she "would call it," she doesn't actually come up with a name, but infers that she could call the figure by a name of her choosing. She infers that she has right and authority to name the figure for her own purposes. By using the word "just", she implies that the “official” name of the object is not as important as knowing that the figure exists, is recognizable, as has a name.

Ms. M routinely did not finish her sentences or thoughts while speaking to the class. This speech pattern was also seen in the interviews and other conversations with Ms. M. In all but a few notable examples of classroom interactions, the implicit messages behind her incomplete statements were clear to me, as an observer. I will discuss this point in more detail later.

The next example shows Ms. M’s use of *they* for different referents. The excerpt begins with a statement in response to a student’s question about the four quadrants of the Cartesian coordinate plane.

Ms. M: Quadrant two. Oh. *We* probably oughta do, let *me* do, let *me* show *you* the quadrants. Since *they*’re talking about the quadrants. That quadrants are, quad meaning how many?

SS: Four

Ms. M: Okay. It’s really weird the way *they* name, number these quadrants. But this is the way. *They*’re always Roman numerals and it always starts here and then it goes counter clockwise. So this one’s one. So this ones gonna be.

This excerpt shows three uses of they with three possibly different references. The first *they* likely refers to the authors of the assignment on which students are working. The third *they* clearly refers to the quadrants. The use of they when referring to objects, mathematical or otherwise, was commonplace in the transcripts and I viewed these as inconsequential and unproblematic.

The second *they* appears to be the most telling of the three. *They*, in this case, refers to mathematicians and those members of a group whose membership grants the authority to name mathematical objects. Ms. M shows that she does not consider herself or her students as having membership in this group by, not only referring to these mathematical authorities as *they* but also, classifying this conventional nomenclature as "weird."

Throughout the observation notes and transcripts, Ms. M positions and repositions herself and students, together and apart, in various groups with drastically varying levels of mathematical authority. The variety in usage of personal pronouns can be clarified if the following questions are answered. For what is Ms. M preparing students? Is it a future as mathematicians or a future of competence using mathematics and an ability to think mathematically? If the answer is the latter, for the vast majority of her students, then what is the most prevailing way mathematical competence is measured? For public school teachers, in light of NCLB and today's air of accountability, the answer is likely the state mathematics achievement test. An analysis of Ms. M's use of pronoun *they* becomes much clearer when *they*, in the above examples and throughout the classroom discourse, refers to authors of the TAKS. If Ms. M's primary motivation is preparing students to pass the state mathematics test, then her pronoun choices can be explained as positioning herself apart from both the students and the test authors.

Figure 2 below shows how the TAKS authors, *they*, Ms. M., *I* or *me*, and the students, *you*, are positioned through Ms. M's pronoun usage. The intersection of Ms. M and the students represents Ms. M's stake in the success and failure of the students after the test results are returned. Ms. M does not have to take the test, but she shares some responsibility with the students. The separation between the TAKS authors and Ms. M and her students is clear as no member of the latter group has membership in the former. This distinction is made perfectly clear in the above examples and many others where Ms. M refers to *they*.

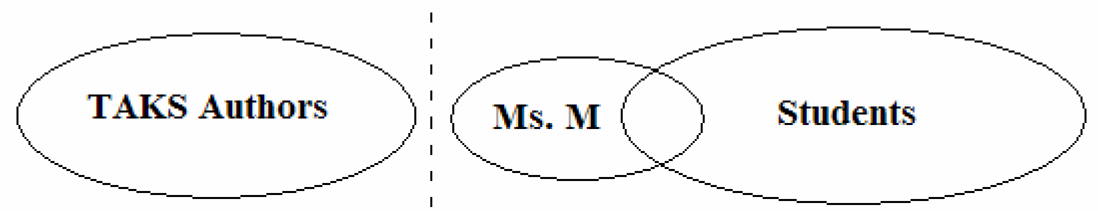

Figure 2. Group Membership Based on Ms. M's Positioning.

When asked to whom they referred, Ms. M responded in a way that confirmed many of the results of my analysis and also shows some alignment between intentions and practices.

> Well, they're the mathematicians. I'm a math expert, but I'm not a mathematician. I would guess just that that's because, it's not my math. I can do it. I can talk to [students] about it. But it's not my math. It's…not my math….Mathematicians, test writers. Sometimes it's test writers. They're gonna give it to you in a specific format.

When asked about what influence referring to *they* had on students and her motivations behind using *they*, Ms. M confirmed some of the results detailed above regarding how she and students were positioned relative to *they*. She reported that they referred to “someone out there, not me, I didn’t write this [assignment, test, or text], I mean, I was not the author.”

Directives and Definitive Statements

This is what you’re gonna do and this is the way things always are. The conflation of mathematical and pedagogical authorities was prevalent throughout each of the classes that I observed. Among the patterns in the discourse of Ms. M were the frequent and related uses of directives and definitive statements. Directives are operationally defined to be statements of direction where the speaker states a necessary condition of action or thought. Definitive statements are operationally defined as statements by the speaker of the way things, mathematically or otherwise, always were, are, or should be. These statements were often used by Ms. M in conjunction with one another. Below, I show some of the many examples where Ms. M used directives and definitive statements with students to direct and define classroom and mathematical behaviors and truths.

In the following statement by Ms. M, both directives and definitive statements are used more than once:

> You *need* to be really careful on, there are some things you can switch out. One half and five tenths are equal ((*emphatically with her hands*)). …These are not ((*pointing to 1/3 and 3/10 on the board*))….If you’re *gonna* interchange numbers, they *have to* be the same number.

Ms. M begins by warning students that they “need to be really careful” when substituting what they think are equivalent fractions for multiplication. The conditional statements help to soften the directive and subsequent definitive as in the statement “If you're gonna interchange numbers, they have to be the same number." The words *gonna* and *have to* strongly imply Ms. M's mathematical authority and are explicit exertions of classroom authority. Students are told, by the same utterance, what they should think and what they are allowed to do.

Although directives were primarily used with the second person pronoun *you*, there was no clear rule that precluded Ms. M from using directives beginning with the third person pronoun *we*. The next statement shows Ms. M's predictive exertion of pedagogical authority in a slightly different way.

> We're *gonna* move on. This is the fifth six weeks…. Um, new grades coming out. We're *going to* work on something different. Take a little break from geometry. We'll come back to similar triangles and Pythagoras, but we're *gonna* take a break.

It can be argued that this statement makes students aware of the direction of the class. Although the words gonna and going to are not stressed, in this statement, the message behind the statement is that the teacher is the one with the authority to direct curricular decisions in this class. Although one could argue that the teacher is the authority in the classroom and is the one that makes these decisions, others might say that students should be given some voice in curricular decisions. In teacher-centered classrooms, however, this exertion of pedagogical authority is likely the norm.

The next example concerns directions related to an assignment on the computer. Students have been given an assignment on the computer and Ms. M was answering a question about one student's confusion concerning the assignment. Ms. M attempts to help the student through the following:

> Well, where are you going? You *have to* go to my website, *don't*, Marisa, you *need* to back outa there, you *need* to be on the main page, the main menu. There it is. Underneath, see, you should be looking at my main menus.

Once again, Ms. M uses definitive statements and directives to exert her authority in an area that falls in an area not as clearly defined as pedagogical or mathematical. The area of technology was an area in which Ms. M's authority was often in question. Ms. M admitted to me that she was not as comfortable working with Macintosh computers as she would have been on a PC. As all teachers and students worked with Macintosh computers in the classroom, many of the students were just as technologically savvy as was Ms. M. Despite this uncertainty of a clear distinction between levels of authority, Ms. M chooses to use definitive statements and directives.

In the next example, Ms. M corrects the way a student pronounced the prime symbol and explains the way things are supposed to be done in this class and in mathematics.

Ms. M: Okay, it wouldn't be read that way, would it? It has that prime symbol. We're gonna call it A prime, B prime, C prime, and D prime. Every time you move a point, you give it a prime. So when it says A', B', C', D', how many points did we move?

SS: Four.

Ms. M: All four of them, right? If we'd only moved one, if we had just moved A, it would just be A prime, B, C, D. We'd call it the same name.

Notice that Ms. M's use of the directive that "it wouldn't be read that way" is followed by the question "wouldn't it?" This question seems especially inappropriate as the student wouldn't have read it "that way" if he knew how it should be read. Ms. M continues by directing the class in the way the prime symbol is "gonna" be read. Further, she uses the definitive statement that "every time you move a point, you give it a prime." The referent of the pronoun *you* is unclear in this case. Ms. M could be referring to the students in this class or a more general *you* that includes all of the mathematical community. In either case, Ms. M exercised authority taking the opportunity to speak for either group.

Statements of Verification

One of the most prevalent themes to emerge from my analysis of Ms. M's discursive practices was an abundant use of statements that verified the validity of students' mathematical efforts. This theme is closely tied to Ms. M's stated intentions of developing students' sense of responsibility in and ownership of mathematics. It also relates to Ms. M's uses of revoicing and echoing. In all but a few occurrences, Ms. M was the one that students came to for validation of their mathematical methods and results. In what follows, I present examples that demonstrate the ways that students relied upon Ms. M for validation and the ways that Ms. M encouraged this reliance through her discursive choices. The examples from the transcripts of this manifestation of authoritative discourse are pervasive and usually brief. As in previous sections, two types of examples are provided, (1) instances of classroom interactions and (2) a list of representative statements made by Ms. M.

This first example of an interaction where a student asks for verification and Ms. M obliges occurs while students have just begun to work in groups. As was typical of the classes I observed, when students worked in groups, Ms. M spent much of her time going from group to group verifying the procedures and answers of each group. In this example, Ms. M is looking over homework papers when a student asks her a question, from across the room.

S: Isn’t one out of four, twenty-five hundredths.

Ms. M: Say it again. Is it what?

S: Isn’t one over four, twenty-five hundredths.

Ms. M: mm hmm.

Rather than having students confirm the answer to this simple question for themselves, Ms. M validates the answer perpetuating their reliance on Ms. M as the one with all the answers. Ms. M continues this exchange by asking questions whose answers would lead the student to the answer of the original problem, but by then, the student already had her answer validated. A different response to the student’s question, and one that could have served to transfer mathematical authority to the student by turning the question back on the student, is “what do you think?” or “why would you think that?”

The next example further illustrates Ms. M’s persistence as the sole mathematical authority in her classroom.

S: Can we round? Like I have 24 and one hundredths. Is it supposed to equal 24?

Ms. M: Yes.

S: (inaudible)

Ms. M: I would do an approximation. I would do it the, um. What did you, what is it? Is it, is that it? Oh, alright yeah. Okay.

In this example, notice that validation came in the form of granting permission and a suggestion. Ms. M confirmed that rounding was "okay" in this case. In other cases involving rounding, Ms. M made it clear to students that they had to get permission to round, as well. The examples regarding rounding show how Ms. M withholds mathematical authority from students.

The following list of statements made by Ms. M, show the types of confirmatory statements she frequently used:

- Yeah. I'll buy that
- It could be two over x. That would be fine
- I'm buyin' it. Definitely true story.
- If it's that whole side, right
- So this shouldn't be here
- No way this is correct. No way that's correct
- you better check again
- Good job, Marisa
- that cannot possibly be that squared
- That's what would be my guess
- I don't think so
- I'm not liking the way that you did it. I'll show you the way I wanna see it.
- Good. Excellent job

Although, there would have been nothing particularly wrong with many of the above encouraging remarks, the pattern of use from which these statements arise show Ms. M's willingness to sit in mathematical judgment of students rather than promoting an environment where students validate their own work and thinking.

It was apparent that students had become comfortable with the classroom norms that Ms. M had persistently established. These are what they saw in the classroom every day. Ms. M who was the self-proclaimed authority, pedagogical and mathematical, in her classroom held all rights and responsibilities to verify the validity of students' mathematics. Who else was going to do it? The class did not use a text that students could check in the back for the answers and the students were not accustomed to validating their own work mathematically. They were, instead, inured to the habit of relying on the teacher for endorsement of each mathematical endeavor, whether during or after each process.

Ms. M's consistent willingness to validate students' mathematical thinking and actions perpetuate an environment where students are dependent on the teacher or some other mathematical authority besides themselves for validation. This relates directly to and serves to contradict Ms. M's proclaimed efforts to foster independent thinking and a sense of ownership of mathematics. How can students simultaneously be responsible for their own mathematical thinking and dependent upon someone else for the answers?

Humor and Sarcasm

There is an abundance of research that reports on the appropriate and inappropriate uses and abuses of humor by classroom teachers. Most agree that humor can be used effectively for various ends, but that teachers should be careful when using sarcasm or irony. Wallinger (1997) argues that the use of humor in the classroom can be an effective way to communicate, establish classroom norms, relieve stress, control conflict, motivate students, and encourage creativity. However, Wallinger asserts that along with embarrassment and ridicule, sarcasm should not be used by educational leaders including, if not especially, teachers. In another study, Dews et al. (1996) examined young children's and adults' understanding of the meanings and functions of verbal irony and found that, with all participants, irony can be used to soften the meanness of criticism. Dews et al. also found older participants appreciated more than the younger children that a meaner remark can also be funnier.

The general tenor Ms. M frequently promoted was light and playful. She regularly used humor with the students. However, the use of humor in the classroom, particularly sarcasm or irony, can be considered a form of authoritative discourse. The teacher, with a multifaceted position of authority, decides what uses of humor are appropriate in the classroom and when she makes the decision to use sarcasm or irony, her classroom authorities are revealed and reasserted.

Below, I provide examples and explanations of the ways Ms. M employed humor and sarcasm in her classroom. As is the case when humor is relayed secondhand, the humor accompanying the story may not translate into a story describing the humorous event. In the following examples, the benefit of a combination of having recent memories and video recordings of classroom events allows me to know that the teacher's statements were made in a humorous manner, despite the possibility that there is no appearance of humorous intentions of the teacher from the excerpts.

Teachers should be willing to laugh at themselves and should aim their humor at the level of their students (Sullivan, 1992). However, teachers should also recognize when humor is inappropriate, i.e., their class should be "taught by a professional, not Bozo the Clown" (p. 72), and should keep humor relevant to the instruction. In the first example, Ms. M used sarcasm directed at herself and kept the humor relevant to the mathematics of the class.

In this example, Ms. M is reminding students of what she feels they should remember about how to divide using long division when both the divisor and the dividend have several decimal places, i.e., long division with decimals. Ms. M instructs the students that the decimal place in both dividend and divisor is to be moved and then digits of the divisor should be "covered up." Enough of the digits of the divisor should be covered so that the remaining digit or digits of the divisor are more reasonable to work with. The reason to make the problem easier to work with is that "I do not know my, my 456ths multiplication tables. If I cover up one. I don't even know my 45 multiplication tables. But if I cover up two numbers, I do know my fours." Ms. M said this with a smile and intended this to be funny, but the intent was to show that to divide without "covering up" would be ridiculous.

Collins (1986) points out that the occasional and appropriate use of humor can be an effective way to avoid or placate crises in the classroom and "uplift everyone present from the mire of a potential breakdown" (p. 20). Collins also warns that sarcasm, a more sophisticated form of humor, is, both etymologically and practically, brutal and "can be as destructive and painful as other forms of humor can be rejuvenating" (p. 20). The following excerpt illustrates one of Ms. M's most striking uses of sarcasm.

At the beginning of one class, Ms. M expressed her frustration with the students because many of them were not completing homework assignments. She explained the way things were, as far as the homework assignments were concerned as follows:

> I say it's homework and you say yeah- right. Then you come in the next day and go- ahh were we suppose to do that for homework? So that's a problem, so get them out and pass them forward… Pass them forward. Don't finish it. That's why it is called homework, because you needed to do it at home.

What keeps this remark from being brutal, destructive, or painful is that it is directed at the entire or most of the class. This does not detract from the fact that the statement was made from a position of pedagogical authority. To strengthen this claim, imagine if a student had made a similar use of sarcasm. Among the benefits of holding a position of pedagogical authority and power is the ability to perpetuate this position through the use of discourse.

According to Capelli, Nakagawa, and Madden (1990), among others (e.g., Dews et al., 1996; Hancock, 2004), both the context and the speaker's intonation may be used for the hearer or the student to understand the meaning of ironic sarcasm. Capelli et al. found that younger children rely more heavily on a speaker's intonation rather than the context of ironic sarcasm for understanding. As the age of the hearer increases, however, contextual cues become just as effective in recognition and understanding of sarcasm. In the next example, Ms. M makes one of the most humorous comments of all the classes I observed. In this example, context and intonation made it clear to the student that Ms. M was being sarcastic. In this excerpt, Ms. M is helping Bernice find the error in the student's work.

Bernice: I added those up and--

Ms. M: --But you know what? Eight minus two isn't four.

Bernice: Oh.

Ms. M: It was gonna be, but then they changed it.

This example also illustrates the way Ms. M has developed a classroom environment where she makes all decisions on mathematical verification.

In what follows, I provide other examples of Ms. M's uses of humor during the classes I observed. The examples are offered to show the general nature of Ms. M's frequent use of humor in the classroom. The excerpts are shown in the same way they appear in the transcripts. Some of the examples are accompanied by comments that also appear in the transcripts.

- I know, I'm getting my act together. It's slow, but it's steady. There you go. See. See my act is almost together.((*laughs to herself as she walks off*))
- Oh, is that right? Ooo, we better talk to her. We're gonna have to send, did you see that? Big fat error. Tsk, tsk, tsk. Ms. Johnson, Ms. Johnson, Ms. Johnson.
- Hm, hm, hm ((*laughing under her breath*)) let me turn the lights on and shine a little light on the situation
- So she's making us actually have to do, like, work. I don't, I don't know about this. We might have to talk to her.
- But I want the work, too, you know, I just love the work. It's so fun.

- It’s the gum. Get rid of the gum and your whole life will be better. Don’t you just love the multiplication? Woohoo. Boy do we feel better now.
- So I would make sure they're both fractions, just nod your head if any of this sounds familiar.
- You got it. One quadrilateral comin’ up.

Not every one of Ms. M’s uses of humor was in a sarcastic manner. Of the examples shown above, several were not sarcastic or aimed at the students. However, the primary way Ms. M utilized humor was through sarcasm.

I considered a teacher’s use of humor, specifically, sarcasm, categorized as a type of authoritative discourse because of the obvious power relations that are involved when someone uses sarcasm. The use of sarcasm in casual conversations and in the classroom, positions the user as exercising power and authority over the other participants in the discourse. Bakhtin’s (1981) description of authoritative discourse as “privileged language that approaches us from without; it is distanced, taboo, and permits no play with its framing context” (p. 424) appropriately describes sarcasm, particularly the way Ms. M used sarcasm in classroom discourse.

Behind the Curtain

Pay all attention to the woman behind the curtain, was one of the messages of Ms. M’s classroom discourse practices. To throw the curtain open and reveal how things are done in the classroom and the mathematics community was a theme Ms. M revisited during every class I observed. Ms. M conducted class as though she had no secrets and all processes, including pedagogical and mathematical, should be made transparent to students. Because teachers do not have the time to explain every possible reason for every classroom decision, Ms. M made numerous omissions. However, a large majority of Ms. M's teaching was in the context of explaining the reasons behind many of the decisions that she made and that students should emulate.

Additionally, Ms. M took many opportunities to share with the class mistakes, mathematical or otherwise, that she or others made. Ms. M professed that this practice is and should be routine for teachers. Her motivation for creating a transparent mathematical environment was for students to observe a realistic model of problem-solving. Ms. M describes below the reasons she highlights the mistakes she makes in class.

> I think kids wonder about stuff like that. They see it and they might not register it, but they wonder about that kind of stuff and especially in mathematics…they need to see what's going on….It's just as important to know what you did wrong as what you did right. And I guess just to model that and to do it, I mean I think that it becomes more natural. Oh I did this, well I was thinking. And sometimes you'll here that in them and I don't know if that's because the way I teach or if it's a natural thing for kids to do. And because I do it too, they just see it as a natural thing.

Below I show examples of the ways Ms. M let the class behind the pedagogical and mathematical curtains. Each example is accompanied by an explanation of the context, meaning, and implication of Ms. M's disclosure to the students. The first example is in regards to the progress report that Ms. M routinely gives out before the grading period ends. In this excerpt, a student asks whether what is being handed out or back to the students is the progress report. Another student is disappointed that the progress report is not what is forthcoming and lets the teacher know. Ms. M feels it necessary to explain the reasons that the progress reports are not yet ready to return.

Notice that Ms. M divulges a great deal of information to the class about when the progress reports would be ready, why they are not yet ready, and what bonus information will be available on them when they are finally handed out.

S: Is that our progress report?

Ms. M: No, I don't have your progress report.

Brian: Dang.

Ms. M: Sorry. Tomorrow. It's a work in progress. (2) It's because I want to put your grade from your, um, release TAKS test on it and we ((*teachers*)) had some little printer glitches and some other issues come up and I'm just, (1) I mean, I have them, but I don't have your release test grade on it and that's what I want on 'em.

In the next excerpt, Ms. M has recently made a "drastic change" in which she began reviewing the previous night's homework problems, particularly one problem in which she could not get students to catch on, despite numerous and varied approaches to the solution. The change occurred after much of the class period was spent with Ms. M trying to drag responses out of the class. Ms. M was reviewing the homework on the computer and trying to re-teach the concepts directly from the homework, which was on a computer document and after becoming frustrated with the class's unresponsive and lackadaisical attitude toward the homework review, Ms. M redirected the class to abandon the computer version and work a similar problem with pencil and paper. In what follows, Ms. M is explaining to a student the reasons why she felt the students were having problems with the original assignment and why Ms. M chose to abandon the computer assignment.

Ms. M: Well, the thing on the computer, the only difference on the computer, which is why I thought it would be easier is here you have the actual points. On the computer, what you were doing was just making it longer or shorter. You were just counting the boxes and doing the side lengths. So if the length ((bell rings)) was five:: (1) and the scale factor was two, then the length would become::? (2) Ten. See I thought it would be easier for you to look at how long the side: was: (1) and then just duplicate this. On the triangle, you had to do the base and the height, but the rest of them were just side lengths.

Another way Ms. M allowed students behind the mathematical curtain was through a process of enculturation into the accepted use of the mathematics register (cf. Pimm, 1987). According to Pimm, “if we are to view mathematics as a language, communicative competence becomes an important consideration, and meaningful communication an overwhelming concern” (p. 6). Ms. M often corrected students’ ways of speaking mathematically and the following examples show evidence of Ms. M’s efforts.

The following interaction involves a student requesting help with a problem. Ms. M asks the student a question about an intermediary step in the problem that requires the student to verbalize a number that has one decimal place. Notice the way the student corrects the way she refers to the number in the problem.

Ms. M: Okay. So I could just cross multiply. Here I'm gonna multiply three times x. And here I'm gonna multiply four and a half by seven. (2) What's four and a half times seven? Let me think. Oh my gosh, multiplying by decimals. This is bad news

S: Thirty (inaudible)

Ms. M: Thirty what:?

S: thirty one point five

Ms. M: What?

S: Thirty one and five tenths.

Ms. M: Oh, thirty one and five tenths. Oh, okay. Thirty one and five tenths. And then what am I gonna do?

Here, the student could have said "thirty one and a half," in accordance with Ms. M's previous statement "four and a half." But using the word "point," as many of us say, is not as mathematically accurate and could lead to future problems, according to Ms. M. There were several occasions similar to the one above where a student would say "point" to represent the decimal and Ms. M would correct the student, forcing them to say the number correctly.

In the next exchange, Ms. M is going over homework problems with the class and is working through a particular problem, which the students had difficulty understanding. Ms. M has the student read the problem aloud as she wrote the dictated the problem on the board.

Tisha: Can we do number four?

Ms. M: On four? What was four? Sh:::

Tisha: (inaudible) ((*reading the problem*))

Ms. M: What, what?

Tisha: (inaudible) ((*reading the problem*)) divided into--

Ms. M: --Divided into

SS: Three and nine-thousand six-hundred seventy-two ten thousandths. ((*Ms. M smiles*))

Ms. M: How many?

SS: Ten thousandths.

Ms. M: Ten thousandths, cause it's how many decimal places?

SS: (Four)

When Tisha read the problem, she verbalized the number 3.9672 mathematically accurately, as she has been taught. Ms. M's smile shows that her efforts have paid off, at least in this instance.

Encouraging a Sense of Responsibility and Ownership

Many of Ms. M's classroom practices appeared to be motivated by a desire to convey a sense of ownership or responsibility to students. One of Ms. M's practices throughout the year was to provide each student with a report of all grades earned by the student for a grading period prior to having to report grades for the class. These reports included a current average for the grading period and grades for each assignment that constituted the overall grade. According to Ms. M, the reason for giving students access to this information was to make them realize that they were responsible for the grade they received and so there would be no questions as to why students received a particular grade.

During an interview, Ms. M shared a story about a student not necessarily in the class I observed, who was having a difficult time remembering to bring a pencil to class. When Ms. M spoke with the student's mother about the problem, the mother asked if Ms. M would be willing to keep a box of pencils at her desk for the student to use during class when he couldn't remember to bring the pencil himself. Ms. M rejected this suggestion but countered to the mother with one that the student use his locker to store a box of pencils. The mother responded that he (the student) just couldn't seem to remember the combination to the lock on his locker. Ms. M said that she was speechless at this point. She asked of me, "Let's see. How is this teaching him to be self-reliant?"

On teaching students to take responsibility for themselves, another way Ms. M reported that she attempts this is by not giving students the answers to questions that they should be able to answer for themselves. On one occasion, a student entered the classroom and asked "What are we doing today?" to which Ms. M responded with "math" and followed with a smile. Ms. M's usual first attempt at avoidance of the questions was through the use of humor and sometimes sarcasm. Uninformative responses like the one above show some consistency between Ms. M's attempts to force students to think for themselves. The answer to this student's question happened to be written on the board and by not answering his question, Ms. M required the student to either find out on his own or wait to find out as the day's classroom activities ensued.

Ms. M admittedly attempted to extend students' sense of ownership to the context of mathematics. She described to me on more than one occasion that mathematics is a series of logical steps and being able to think mathematically is "logical thinking, from one step to the next." In reference to students sense of mathematical ownership, she "foster[ed] it in a really horrible way by forcing [students] to do their own thinking." Ms. M's account of the way she fostered students' sense of responsibility and ownership was not, however, consistent with her teaching practices, particularly her discursive choices. In what follows, I will provide several examples of classroom interactions that illustrate this point, each accompanied with a description of the surrounding classroom and mathematical context.

The first example is not mathematical, in nature, but is related to the game of the classroom for which Ms. M sets and enforces the rules. In this excerpt, Ms. M is collecting completed quizzes and notices that a student has completed his quiz using a pen instead of pencil.

Ms. M: This is in pen. ((*the timer/buzzer goes off and Ms. M steps back to address the class*)) Please make sure your name, date, and the class period is on your, uh, calculation worksheet and go ahead and pass them forward, please. ((*taking up the warm-up*)) Thank you, sir.

S: (inaudible)

Ms. M: Thank you sir. I know, haven't you seen me write everything, quit, don't click it, thank you. Haven't you seen every time I hand a, hand a paper back to you I keep writing "pen" and a question mark? How come you keep doing your math in pen? How come you keep doing your math in pen? *You can do it in pen, but it's gonna cost you ten points.* It's like a toll. You know, like a toll road or a fee to get into the basketball game or something? ((*finishes taking up the quizzes*))

This example shows that Ms. M has laid out clear rules of the classroom game and that there are penalties for those students who break the rules. The rule highlighted here is that students must complete mathematical work in pencil, when working on paper assignments. The student in this excerpt has shown and continues to show that he is not willing to abide by this rule through his persistent use of a pen. Ms. M then provides the student and the class with a choice of following the rule or not. By providing the choice and explaining the consequences of their choice, Ms. M encourages students' sense of authority and ownership for their actions. They can act agentively against the existing structure

The next example shows Ms. M helping a student to plot points on the coordinate plane. In this excerpt, Linda responds to a question with "I don't know." Ms. M treats this response as an attempt to avoid the question. Instead of telling Linda the answer, Ms. M tries to convince her that she does indeed know how to plot the point.

Ms. M: … Did you graph them all on your plot? You need to put them on here. ((*taps with a ruler on Linda's graph*)) That's what the mission is.

Linda: how?

Ms. M: How? How do you plot negative one, one ((*the point (-1,1)*))? You start at the origin. Go negative one. Is that right or left? Negative one.

Linda: I don't know.

Ms. M: *yeah you do. You did all kinds of graphing before*. Negative one. Right or left?

Linda: left.

Ms. M: How many?

Linda: One.

Ms. M: And then which direction?

Linda: I don't know.

Ms. M: Well, it's positive one. What are your choices?

Linda: I don't know which way, I know that part. I don't know. This confuses me.

Ms. M did not accept Linda's "I don't know" response and, instead, countered it with "yeah you do. You did all kinds of graphing before." Ms. M then proceeded with her line of questions, which became more specific than the original. By not accepting Linda's initial response, Ms. M suggested to Linda that she had ownership of her mathematical learning and could not avoid responsibility through avoidance. This excerpt ends with more avoidance by Linda.

Although more of the transcript is not provided here, the transcript continues showing that Ms. M leaves Linda to begin a discussion with the class on plotting points on the coordinate plane. Through this class discussion, Ms. M makes use of other students' knowledge to remind Linda that she did, in fact, know how to graph ordered pairs. By not letting students "off the hook," Ms. M show some consistency in her claims that she make students think for themselves. In this case, however, it would be more accurate to state that Ms. M has not done the thinking for the student. Instead, she had other members of the class think for Linda.

In the next example, Ms. M encourages students' sense of responsibility for and ownership of mathematics by encouraging students to go ahead in the lesson to a problem that, as a class, they had not discussed. In this example, the class is discussing a PowerPoint presentation of applications of the Pythagorean theorem. The presentation is sequenced so that students first learn how to find the length of the hypotenuse of a right triangle given the two other side lengths, i.e., the legs. A slightly more complex application is using the theorem to find the length of one of the legs given the lengths of the other leg and the hypotenuse.

Ms. M: Okay, good. Then try that one for practice and then we'll check it out. ((*to James*))…Whatchya got? ((*looking at James's paper*)) Yeah. Yeah you can try to see, if you wanna try to see if you can calculate the leg. ((*to class*)) If you're a real stud muffin, you can go ahead and try to calculate the leg, but don't go on. Check and see if you're right before you do anything else. If you get the, the hypotenuse, the next one is going to be calculating the leg.

By having students make an effort at finding the length of a leg and extend what they have just learned, Ms. M is encouraging students to take ownership of the new mathematical knowledge. By stating that students should move on only if they thought they were "real stud muffin[s]," she affirms in students that if they are willing to take risks, they must necessarily have qualities that are appreciated by Ms. M and likely others.

By explicitly stating the directions to move ahead to the next application of the theorem, Ms. M exhibited her control over the direction of the class. Notice that Ms. M also tells the students that calculating the leg is as far as they can go and not to go any further without checking their answer. Checking the answer, in this case as well as in most, involves comparing the answer the student obtained with the right answer as provided by the teacher. Ms. M is reinforcing the notion that she holds a position of higher authority than others in the class. Her classroom authority is implied through directions of what the students can and cannot do and her mathematical authority is implied by her having students check their answers with hers.

The following exchange occurred a few minutes later, during the same class when Jeffrey asks if it makes a difference whether or not he solves the Pythagorean theorem for the leg length before the other variables have been substituted with specific numbers.

Jeffrey: Is it alright if we swap the letters around and then fill in the numbers?

Ms. M: Swap the letters around and then fill in the numbers. Oh. Yes, that's fine. That's fine. Yes, that's fine.

This exchange is an example of a potentially, mathematically agentive act that was not allowed by either party to take place. Jeffrey was not taught that he could solve the equation for one of the variables and then substitute in values for all other variables. Instead, he considered that this method was a likely better and more efficient way that made more sense to him and he probably used this method upon first attempting the problem. Instead of using any mathematical authority that he considered himself to possess to verify the validity of his new method, he went directly to the teacher, someone whom he knew to possess the mathematical authority, to validate his method.

Ms. M responded in such a way contrary to her claims of making students think for themselves. Had she been true to her stated intentions, she could have turned the question back to Jeffrey by asking some form of “what do you think?” She could have had Jeffrey show the class his new method and had the class decide on the validity of the method. But by answering the question directly, regardless of the answer, she chose to perpetuate the type of affirming questions that she claimed not to answer.

Jeffrey was a student who, from my first observations I could tell, was outstanding. Whether Jeffrey stood out in ways that are, by most school standards, considered good or not, I could not tell at first. After continued observation and discussions with Ms. M, I learned that Jeffrey possessed the potential to be both academically outstanding and behaviorally challenging. During my second observation, I was seated in a student desk on the far left side of the class, at the edge of where students were seated. Jeffrey was called out of class to go to the assistant principal’s office, for whatever reason. Upon returning, Jeffrey had to walk behind Ms. M, who was standing in front of the class at her computer projecting an assignment on the board, to return to his seat. As he passed behind Ms. M, he looked in my direction but at Brian, a student that was sitting near me, and made an obscene gesture with one finger at Brian. My initial thoughts about Jeffrey were strengthened, if not confirmed.

I also learned that Jeffrey was the most misplaced student in the class and could, with a lot of effort, succeed in Algebra 1, the class a level above Pre-AP eighth-grade mathematics. Ms. M stated the following to me in response to a question about Jeffrey:

> Jeffrey is right at the tippy top of Pre-AP, but not good enough to be algebra 1….Jeffrey could do this with being absent three times a week, two days a week…but he's not good enough to be in algebra 1…or, he doesn't wanna take Algebra 1, so he didn't test well enough to get in it. So…he likes being a big fish in a small pond. He's not ready for the big pond yet….There's not a question in my mind that he could, he should be sitting in my Algebra 1 class, but he doesn't wanna do homework or this, or he had a bad testing day. But for whatever reason, he's in a class a level too low for him…He's probably the smartest kid in that class. There might be, he's in the top three of that class, and he really should be sittin' in an Algebra 1 class.

Jeffrey, along with other classmates, may have acted in mathematically agentive ways throughout the year and under the instruction of Ms. M, but the concentration of this study was on the ways that Ms. M used discursive strategies in her classroom to encourage and foster such agency.

Ms. M was very forthcoming in her hierarchical authority of classroom events and mathematics, relative to the students in her class. She let the students know through discursive practices that she was a greater authority than were they, despite her admissions of an intent to foster independent mathematical thinking in her students. By creating an atmosphere of mathematical dependence on higher authorities, students were not encouraged or allowed to do or think for themselves. Throughout this chapter, I have shown the ways Ms. M’s discursive practices worked to inhibit, rather than support, students’ mathematical agency.

In order for students to regularly act in mathematically agentive ways, they need to have an atmosphere where they are expected to think for themselves. Although Ms. M claims to make students think for themselves, repeated student questions for specific mathematical permissions and directions show that these claims are not accurate, i.e., Ms. M’s intentions do not correspond to her teaching practices.

The results and subsequent discussion and conclusions from this and the next chapter are based on one of many perspectives with the aim of analyzing practices of effective teachers. Because there are many ways to determine the overall quality of a teacher and many facets that constitute pedagogical practices, I offer my perspective by which one of these facets may be assessed.

CHAPTER V

DISCUSSION AND CONCLUSIONS

Summary

An overarching goal of this study was to show to what extent reform-oriented mathematics teachers' practices were consistent with their teaching goals. Toward this end, I conducted a case study of one middle school teacher with the purpose of discovering the level of agreement between the teacher's intentions and practices. Did this teacher do what she intended to do? Did her teaching philosophy agree with her teaching practices?

The conclusions supported through this research and reported in this chapter address two research questions: How do mathematics teachers' classroom discourse practices reflect their willingness to (1) share mathematical authority amongst their students and (2) accept solution methods and explanations divergent with their own?

In this chapter, I begin with a discussion of the ways in which the results of the last chapter inform and answer each research question. I continue this chapter with implications and suggestions for teaching practices. I conclude this chapter and dissertation with the limitations of this study and suggestions for future research in the area of classroom discourse.

Discussion of Research Findings

In the discussion of the results, I will discuss the ways in which each of the themes from Ms. M's teaching practices reflected her willingness to share mathematical authority and develop mathematical agency, as evidenced by a willingness to accept students' alternative solutions and explanations divergent from her own. The first of the themes discussed below involve the relationship between teachers' discursive choices, particularly the use of authoritative discourse, and positioning in various (mathematical) communities. The next theme relates authoritative discourse to creating a sense of dependence in students. The final theme relates the use of authoritative discourse to a perpetuation of a hierarchy of mathematical authority.

Through discussions and interviews with Ms. M, I found that both her willingness and intent to share mathematical authority with students were present, but required some qualification. Ms. M was the self-appointed mathematical authority in the classroom and, according to her, the students were well aware of this hierarchy of authority. Her willingness to share mathematical authority was expressed by her desire to foster a sense of responsibility and ownership of mathematics in her students. Also, Ms. M expressed that she tried regularly in class to teach the accepted ideas, methods, and language of the mathematics community. As a member of this community, Ms. M felt that a large part of her job involved providing membership benefits and responsibilities to her students.

The Mathematics Community

The first theme to emerge as a result of my analysis was that a teacher's use of authoritative discourse strategies can serve to define membership and levels of membership in the mathematics community and other communities in which she and the students belong. Although granting membership in the mathematics community to students may be the desire of teachers, the different communities to which students may belong are numerous and membership may vary. Some of the positionings that occurred as a result of Ms. M's discursive choices are shown in Figure 3.

Figure 3a shows a relationship where Ms. M and the students are together but separated from the mathematics community. This is the relationship that was produced by many of Ms. M's uses of *they*. Figure 3b shows a positioning of Ms. M and the students as together in the struggle to learn and do mathematics as a part of the mathematics community. This positioning often came about with Ms. M's uses of inclusive third-person plural pronouns such as *we*, *us*, and *our*, e.g., "I think it's maybe trying to learn it on the computer might have confused *us*." Figure 3c shows a similar positioning as the last except here there exists less of a cohesiveness between Ms. M and the students. She positioned herself and the students as part of a bigger mathematics community, but with distinctly different roles and responsibilities. With this positioning, much of Ms. M's pedagogical authority was exerted as mathematical authority. The overlap of the two groups, Ms. M's and the students, represents Ms. M's responsibility for the mathematical successes and failures of her students. Figure 3d represents a positioning of the students as outsiders with respect to the mathematics community, of

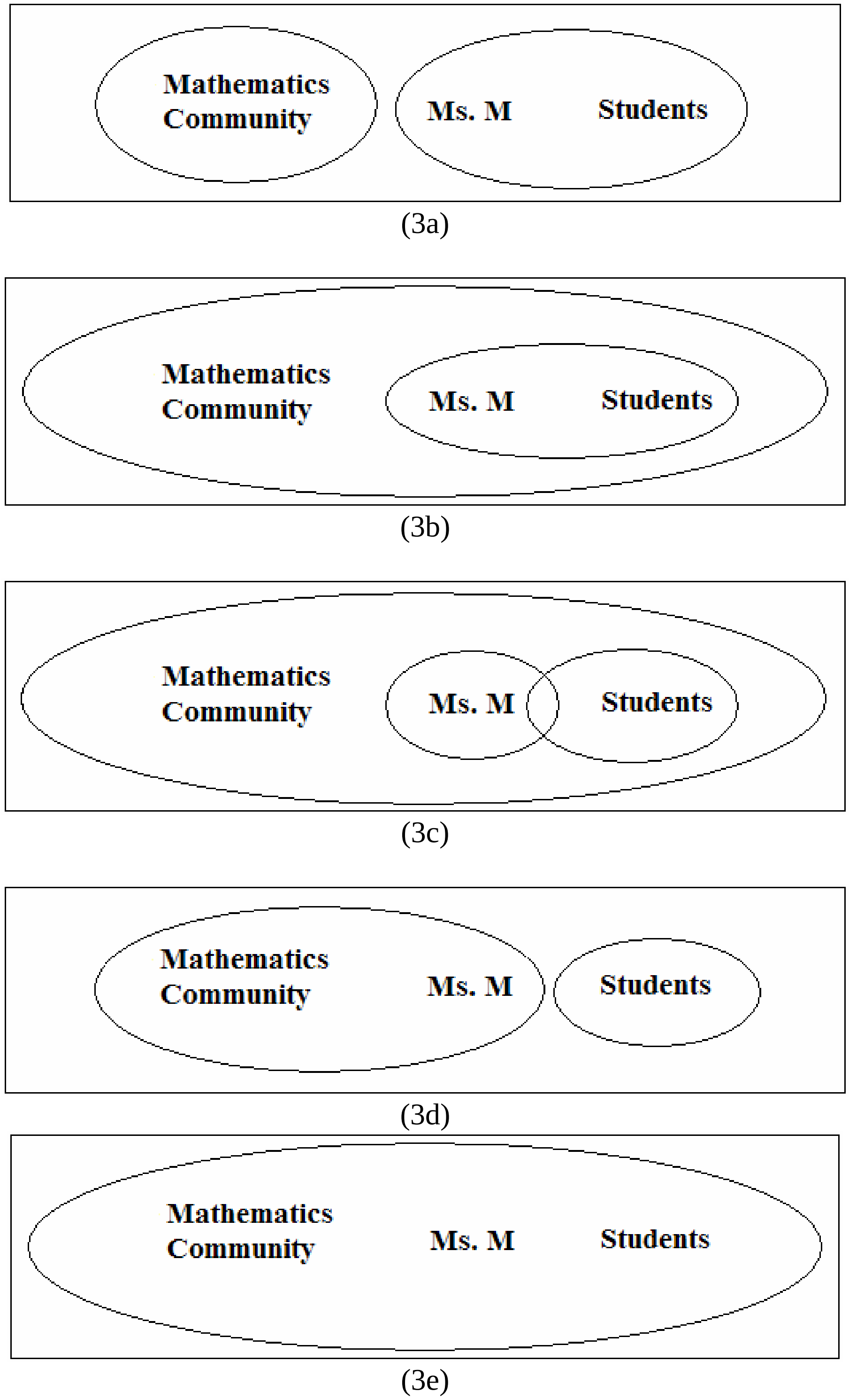

Figure 3: Positioning Relative to the Mathematics Community.

which Ms. M is a member. This positioning occurred when Ms. M used we, us, and our to include herself in a group of mathematics experts who should be emulated. In Figure 3e, students are positioned in as members of the mathematics community. In this representation, Ms. M has an equal stake in construction and learning of mathematics. Although this version was rarely observed, I present it as an alternative in which mathematical authority and agency are most likely to be shared and fostered, respectively.

One of the most prevalent discursive choices Ms. M made in her class was her varied and sometimes inconsistent choice of personal pronouns. Through Ms. M's choices of personal pronouns, membership in the mathematics community was shown to be fluid, that is, Ms. M held the authority and exercised the power to sometimes include students in the mathematical community and sometimes keep them out. Ms. M's own responsibilities as a permanent member of the mathematics community varied according to her personal pronoun choices. These different levels were expressed as she positioned herself relative to text and test authors, administrators, mathematicians, answer keys, or other "theys" in the mathematics community. However, relative to the students, Ms. M consistently held a position of mathematical authority. Even during those times where she deferred to "they" as a more powerful authority, she positioned herself as able to speak for the group that included the students and herself.

Inigo-Mora (2004) describes various notions of identity and community. She asserts that "a school community shows how vague the limits of a community can be. One student, although technically a member of this community, may feel that s/he does not belong to that community because s/he does not share its values" (p. 29). Teachers can assert the power to define various communities through discursive practices. A broad category, such as the "community of students" or "community of mathematics learners", may not be specific enough to describe the changes that rapidly occur throughout the school year, the semester, the grading period, the time between tests, the school day, or even the class period. Divisions within a broad category of "students" are both beneficial and necessary to discuss how students are positioned with respect to different communities.

Having membership in the mathematics community carries certain responsibilities as well as certain benefits. These responsibilities include doing mathematics in a manner that is not inconsistent with particular axioms. This is not to say that creativity is discouraged, but that any creativity must remain within certain bounds. One of the benefits of membership is the authority to justify and validate mathematical procedures and conclusions. Through authoritative discourse practices, Ms. M afforded students the responsibilities without giving them the benefits.

For a teacher who positions herself as having the authority to grant membership, the use of authoritative discourse may be second nature and not misalign her practices and intentions. However, if a teacher's objectives include granting students membership into the mathematics community, by sharing mathematical authority and developing mathematical agency, then authoritative discourse practices are likely to inhibit these objectives.

A Culture of Dependence

The second major result of the analysis is that Ms. M's use of authoritative discourse created a classroom culture of dependence that was apparent in her students' questions and comments during the classes I observed. By directing students' actions and preferred ways of thinking, Ms. M created a sense of dependence in students on the teacher for validation of their actions and knowledge. This dependence was reproduced through Ms. M's persistent use of confirmatory statements. Because she established herself as sole proprietor of mathematical authority, students looked to Ms. M as having "the answer." The result of a classroom steeped in authoritative discourse was the production and reproduction of a classroom culture where everyone's mathematical actions and knowledge are dictated by some other authority besides themselves.

On many occasions, Ms. M showed that, as much as her students depended on her for validation, she depended on others as well. In this culture of dependence, mathematical knowledge and solutions were not considered authentic unless someone could authenticate them. Through an extensive and persistent use of various forms of authoritative discourse, Ms. M inhibited a professed intent to foster a sense of ownership of and responsibility for mathematics in students.

Authoritative discourse practices of teachers, such as directive and definitive statements, can inhibit students' independent thoughts and actions, unless these statements take a form such as "you must think for yourself," "you are going to argue that your own answers are mathematically valid," or "it is always the case that the explanations of your results are more important than the actual results." These potentially productive directive and definitive statements, which would have been aligned with her stated intentions, did not surface in Ms. M's discourse practices in the classroom.

Utilizing particular authoritative discourse practices can result in students' dependency on a higher authority, which may or may not be the teacher, to define what is mathematically correct and acceptable. This result is consistent with Prevost's (1996) assertion that students should rely less on the authority of their teachers to tell them if they have the correct answer or if they are solving the problem in the correct way. This dependency is likely to preclude students from sharing in mathematical authority. Such a dependency also can and likely does suppress mathematical agency in students. If students must depend on someone or something for mathematical validation, when do they learn to determine if their mathematical constructions are valid or acceptable? How do students know if they are right or if they are thinking correctly without checking the back of the book or taking their ideas to their teacher? If independent mathematical thinking by students is a goal for mathematics teachers, then discursive choices that include authoritative discourse may be counterproductive.

As was mentioned previously, one of the constants of all mathematics classrooms is that the teacher is the authority in the classroom. The authority of the teacher can take the form of pedagogic action or mathematical knowledge. How s/he chooses to enact or share aspects of these authorities is not a constant. For teachers who wish to share mathematical authority with students, to develop a culture of mathematical independence, the role of the teacher should not be final decision maker. Instead, the teacher should realize a role as negotiator between students and the mathematical community or mathematical knowledge (Richards, 1996).

The Hierarchy of Mathematical Authority

Through Ms. M's uses of authoritative discourse, a hierarchical relationship between her mathematical authority and that of students was created and perpetuated. The difference between levels of mathematical authority of students and Ms. M were regularly revealed to students through authoritative discourse practices. Within the hierarchical structure created by Ms. M, it was conceivable that an individual at any level have mathematical authority. However, Ms. M hindered the sharing of mathematical authority among students and discouraged mathematical agency primarily by directing all mathematical behaviors and making all decisions on mathematical validity.

A hierarchy functions in the greater mathematics community to the extent that for each individual, possibly excluding research mathematicians at the top of their subfield, there is always a higher mathematical authority that can and will make the "final decision" to confirm mathematical validity. With such a system in place, very few individuals have autonomy and are encouraged to act in a mathematically agentive manner.

Does a consideration of a teacher as having the power to share or distribute mathematical authority actually serve to perpetuate this proposed and observed hierarchy of mathematical authority? Is the notion of sharing authority problematic? An idealistic view of the world, particularly, the community of mathematics, is one that sees all members as having equal power to contribute their performances, ideas, and reasoning. A hierarchical relationship between individuals and groups of members of the mathematics community does exist and can be seen in classrooms, meetings of mathematics faculty at the secondary and university level, tutoring situations, or meetings or conferences of professional organizations, among many others. Another example where this relationship clearly shows its existence is with a student completing a homework assignment. When the student finishes a problem or the assignment, s/he may likely check the answers in the back of the book or turn the problem into her/his teacher to be graded.

A reasonable and preferable alternative to this system may be for students to return with their homework to a classroom where the confirmation of the validity of the mathematics is determined by the classroom community, including students and the teacher, through argumentation. With this alternative method of mathematical validation, a hierarchy of authority can/does exist but is not imposed on students.

Authoritative discourse blurs the lines separating pedagogic and mathematical authorities. For teachers who are comfortable with maintaining a disparity in levels of mathematical authority, such as those between themselves and their students, the use of authoritative discourse in mathematics classroom communications may well serve their intentions. However, for teachers whose objective is the sharing of mathematical authority and the development of mathematical agency, a reproduction of a hierarchical system of mathematics authority based on one of pedagogical authority can be unproductive and misaligned with their intentions.

Implications and Suggestions for Teaching Practice

My analyses of one teacher's uses of authoritative discourse reveal the complexities and difficulties confronting teachers in mathematics classrooms. There often exist multiple interpretations of any particular utterance of a teacher or student. Therefore decisive conclusions about a connection between the devices that can be identified as authoritative and an intention to control mathematical authority should not be made without considering the source and context. My intentions through this study were to observe and analyze the ways words were used in the classroom and use the analysis not to criticize teachers but as a mirror for teachers to reflect on their practices. If teachers do not see their practices reflecting their intentions, then changes in their discursive choices may be in order.

Through this research, I have accentuated the tendencies of one teacher to control mathematical authority through the employment of both subtle and not so subtle discursive devices. My analyses suggest that teachers, who use authoritative discourse in their classroom in a similar manner as the teacher in this study, may position their students in such a way that mathematical authority is not shared with students and mathematical agency of students is not developed. In order to share the authority in a way that confirms mathematical explanations, teachers may reconsider positioning themselves, the text, and the often referred to "they" as the only validating sources in their classroom. Students, individually or collectively, can and should be asked to validate emerging mathematics if teachers want to develop mathematical agency and promote ownership.

One common practice teachers might use to bring students into the expert mathematical group to which teachers belong is to explain to students the validity of their argument or solution. However, students should struggle to find and justify solutions on their own rather than the teacher either giving the answer or validating a solution (von Glasersfeld, 1995). Gergen (1995) argues that language in the classroom is nonsense until it becomes communication between two or more people, e.g., teacher with student(s) or student(s) with student(s), and sense is made from the language. Thus in order to achieve a wider distribution of mathematical authority, teachers should require that students confirm their own ideas or come to an agreed upon consensus with their classmates.

Prescriptions of how teachers should speak, e.g., what pronouns teachers should use, are, by no means, the end of these conclusions. But, at the same time, an apolitical, watered-down descriptivist view of teachers' classroom language would be naïve (Pennycook, 1994).

There are potential problems in drawing general conclusions and making pedagogical suggestions based on the analysis of one teacher. The suggestion I make here is that, teachers should first closely examine their intentions in light of current beliefs about teaching based on educational research. Once the teachers better know themselves and their objectives, they should examine their teaching practices to determine consistency between their intentions and their practices.

Do teachers intend to share mathematical authority or do they think that they know what is best for their students and are the ones whose job it is to tell them? Do teachers really intend to foster mathematical agency? What would a class in which all students felt encouraged to be mathematically agentive look like? For teachers whose intentions are, in fact, the development of mathematical agency in and sharing of mathematical authority with their students, their discursive choices should be made carefully. In particular, they should use authoritative discourse deliberately and carefully and should be reflective on the ways their discursive practices affect their students' perceived level of mathematical authority and agency. This type of reflection should be commonplace among teachers and could lead to better alignment between ends and means of teaching.

Further Research on Classroom Discourse

From the beginning of my involvement in this research process, I developed a fascination and curiosity with discourse analysis and the potential that various types of discourse analysis held for explaining teaching processes. I made particular choices based on relevant research in the area of authoritative discourse and its relation to classroom communication practices. These choices were not arbitrary, but could have been different. For example, I would be interested to learn whether the same conclusions would have been reached or enriched through an analysis of Ms. M's discourse utilizing Politeness Theory (Brown & Levinson, 1987) analyzing the teacher's discursive choices as politeness strategies to demonstrate control.

In order to make more applicable the conclusions from this research to other middle-school mathematics teachers, a comparison study of teachers found to have philosophical intentions similar to each other could be a next step in research. By comparing different teachers' uses of authoritative discourse, one could more easily make generalizations as to the ways these discursive strategies align practice with intent.

As both a practical application of this and related research on classroom discourse and a new vein for more research on this subject, I suggest a professional development opportunity where middle-school mathematics teachers could first learn how to identify their teaching goals and orientations. These teachers could then be observed and video recorded in their natural classroom setting, after which they would return to analyze their teaching practices based on recognition and explanations of their authoritative discourse strategies and enactments. The teachers could then reflect on the alignment of their goals and the way those goals were or were not met through their use of authoritative discourse.

Authoritative discourse is present in all social settings including but certainly not limited to middle-school mathematics classrooms. To what extent is authoritative discourse present in higher or lower level mathematics classrooms? To what extent is authoritative discourse present in classrooms of other school subjects such as science or social studies? How do elementary teachers, who teach many different subjects to the same group of students, make use of authoritative discourse practices and how do these utilizations affect a transfer of intellectual authority to students at the elementary level? These, and many others, are questions that could be investigated and whose answers could help in the improvement of teaching practices in mathematics and beyond.

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APPENDIX A

DESCRIPTION OF TRANSCRIPT NOTATION

Transcripts of all classroom interactions follow a system that was intended to provide more detailed account of the classroom events than a simple version that translated verbal language into written. Below, I describe the rationale for the transcript notation. I follow with a description of the facets of Jeffersonian (Jefferson, 1984) notation which I employed in the transcripts. I also provide excerpts from the transcripts, to illustrate my notational usage accompanied by explanations of each of the uses.

The overarching goal during transcription was finding a reasonable nexus between “accuracy, readability, [and] usefulness” (Romero, O’Connell, & Kowal, 2002, p. 620) of the transcribed data. Romero et al. found when different experts transcribe the same data, multiple differences using the same notation are likely and that these fluctuations in notational use are virtually unavoidable. Considerations for the usefulness of the transcripts, with regard to the intended type of analysis, led me to a primary goal of obtaining an accurate account of the words used by all participants. Secondary to this goal was accurately capturing the ways those words were spoken including pauses, overlapping speech, sound prolongations, audible exhalations and inhalations.

The only speakers in the transcripts were the teacher, Ms. M, the intern, Laura, and students, either their name, S, or SS, where SS means more than one student was speaking at approximately the same time. Because the teacher was the focus of the study and, therefore, the only speaker with a microphone, many of the students’ utterances were unintelligible. These utterances were notated with (*) where * was replaced by either the word “inaudible” or an approximation to the utterance.

Pauses between utterances by a single speaker within a single statement or between speakers was denoted by a (*), where the * was the number of seconds of the pause rounded to the nearest second. Natural speech is not the same as written speech and most of us do not speak as though we are reading a script of neatly written prose. However, commas and periods were sufficient to record pauses shorter than one second.

Overlapping speech in the discourse is indicated by the use of a double hyphen (--). This notation appeared at the end of the utterance of the speaker who was interrupted and at the beginning of the utterance of the speaker who was the interrupter. I opted not to use strict Jeffersonian notation for overlapping speech because I did not see its utility with respect to my analysis.

Sound prolongations were marked with colons following the syllable which was prolonged. Prolonged syllables that ended in consonant sounds such as *right* were marked after the word (e.g., right:::) rather than directly following the prolonged sound (e.g., ri:::ght). It is reasonable, in this case and many others, that the vowel sound of the word *right* would be prolonged rather than the ending consonant sound. The number of colons used indicates the length of the continuation of the sound.

Additionally, when speech was not comprehensible or unclear, *inaudible* or an approximation to the speech was placed in single parentheses, e.g. (inaudible). When additional comments regarding verbal and nonverbal activity were warranted and/or helpful, I enclosed the italicized comments in double parentheses, e.g., ((*teacher walking around the classroom*)).

Commas were used to indicate natural pauses in speech in a similar way they are used in written language. Periods were used to indicate a stop in the spoken sentence. Due to the abundance of incomplete sentences or thoughts spoken by Ms. M, periods and commas were often used in a similar manner. Question marks were used to indicate a rise in the speaker's intonation, signifying the end of a question. Finally, (hhh) was used to mark an audible exhalation by the speaker.

Below I provide a sample from the transcripts in which each of the above mentioned notational conventions appear at least once. Following each of these conventions are comments delimited as follows, {**explanation of notation**}.

Sample Transcript

Sheila: Okay, so first initial-- {**double dash indicates that Sheila was interrupted by Ms. M}**

Ms. M: --I told you guys to do it, okay, hold on, sh, sh, sh. I'm comin' around. I'm comin' around. God bless you. This is your log in and that's your password. Is that what the problem was? It should be, when you do it, it's the frog one, it's the very first one listed is what it is. Mr. Williams. Now I would appreciate it if, when you see it, if you didn't remember it, you write it down somewhere. This is your sign in and that is your. Okie doke.

S: password. {**S indicates a student whose name I was not able to discern from the video recording, If the student's name was known, then it was used in the transcript**}

Ms. M: yeah, when it comes up, it should be a little frog, right? It's like little frogs all stacked on each other or something? There's something about frogs.

S: the frogs are (inaudible)

Ms. M: Sh::::: {**elongation of the sh sound where the number of colons represents the length of the elongation, measure relatively**}, okay, sh::::::::::::. Sh:::::. Here's your log in and there's your password ((*continuing to walk around with the passwords*)) {**indicates my comments on surrounding events**}(10) {**indicates a ten second pause**} Log in, password. (14) Log in, password. (27) yeah, that's it.

APPENDIX B

EXAMPLE TRANSCRIPT

0:37

((*Ms. M sets the timer for the students to do their warm-up*))

Randall: The similar triangles, the similar figures from yesterday (inaudible). Was that homework?

Ms. M: well, what you can do is, I'm gonna be giving the students who were not here yesterday those notes, so while they're doing that, if you want to finish it, that is fine. ((*Ms. M is walking around, handing out papers*))

Randall: I've already finished it (inaudible)

Ms. M: Oh. Then the other thing that I did was I loaded up some um, (2) there's three (2) gizmos. So you can go to explorer learning and do Similar Figures A first. Then do Similar Figures B. Then if you have <u>time</u>, while I'm still giving them the notes, you can do Similar Figures, er, Similar Polygons. (7)

S: (inaudible)

Ms. M: yes, you are. (6) Sir? Oh I thought you were raising your hand. Sorry.(5) God bless you. (17) Uht, I don't know how that one. (2) Oh::, you know what? This might be a bad thing. (3) I'll tell you in a minute. (84) (cause I'm asking you, where is what they're asking you for)

S: (inaudible)

Ms. M: Yeah, you know what happened was, we, uh, you guys, gave this to you that day we were doing the review and I was sitting back there trying to grade them and: there was a lot that was missing and so I ended up drawing a bunch of lines, and I thought, you know, instead of giving 'em zeros, what I'm gonna do is I'm just not gonna grade these and return them. Ma'am? ((interrupted by a visitor)) (5) and then I put them in the box and I never returned them to you. I never passed them out. So what happened was, um:, then some other people turned some more in and I graded those ones and so, I'm gonna give you instructions on all of them. Because what a lot, all, if, <u>if</u> it has my writing on it, if it has my writing on it, you can fix it, finish it, correct it and I will regrade it up to a hundred. It should have, and I'll be able to tell whether it's like, um, you know, um, I mean I'll be able to tell. I, I would have written on it, run through it, put a big question mark on it, something like that. If you fix it and turn it in, I'll grade it up to a hundred. If you have yet to turn it in to me, and I have no writing on it, then you may turn it in but I, but I don't know, there's no way I can give you more than a 70 on it. (1) okay? So (2)

SS: (inaudible)

Ms. M: Here you go. Am I gonna do what?

S: (inaudible)

Ms. M: We're talking about the translation, reflection, and trans, uh::::,--

S: --(inaudible) we can turn ‘em in again?
Ms. M: Yes. Yeah, because those grades were pretty low. ((*to Mandy*)) you need to take this home and get it signed and return it to me (3) Now this is what:, (1) wait, and let me talk about everybody’s cause I’m still handing them out. The, the next thing, I have not, con, uh, I’m not sure that everybody, if you were absent on Monday and have not yet taken that test, that’s why you’re, I’m not handing the test back and also those of you who said you were gonna come in and finish ‘em, I never saw you all. You took it on Monday. Today’s Thursday. We’re a little late. So I’m gonna go ahead and grade those (2) Um, and so: Uh, but you need to, but I can’t return them until everyone’s made it up. So if you haven’t made it up yet, you need to get to me. Get with me.
S: When are we supposed to come in? Cause like, Monday--
Ms. M: --before or after school--
S: --we asked if we could and the sub said we couldn’t (inaudible) the TAKS thing, and then yesterday (inaudible)
Ms. M: yesterday morning, I was here and you guys were here before school and I was here after school and then this morning, I was here before school
S: Yeah, I was here. I couldn’t get here yesterday morning but::
Ms. M: yeah, see normally to finish a test, you have that day after school and the next day before school but when the class meets again, then it’s too late. (2) okay so then to me it’s like it was yesterday morning, yesterday after school, this morning, ((*hands out as if to say ‘what do you want me to do about it*)) I was here Tuesday, yeah, I was here after TAKS day, after TAKS
S: I could never get here (inaudible)
Ms. M: then what difference would it make? (4) No matter how, you see what I’m saying? If you can’t ever get here, what difference does it make?--
S: --(inaudible)
Ms. M: No (1) it’s too late now.
S: (inaudible)
Ms. M: Then you need to hurry up and get in and take it.
S: (can I have a pass?)
Ms. M: Yep, you'll have to make sure you get with me to get a pass (5) ((*continuing to hand out papers*)) Okay, so here’s the last one. The other thing that you have in front of you is your individual: TAKS (hh) (6) it’s enough, okay ((*to the complaining student who is continuing to talk about it with another student*)) (3) your individual release TAKS report, this is what I did, I stapled your: tutorial strip, that’s what it’s called, to the back of it, um, for the most of you. Now, what I, what it is, is you should have, there are six objectives the TAS, TAKS test is testing. Number one is numerical, uh, numbers, operations, um I forget no exactly what it says. Number, operations, and quantitative reasoning. Those are the ones where it’s, you know, this is the total bill::, this person paid two dollars and 75 cents::, this person paid a third of it, this person paid, sit down and wait okay? ((*to Sheila who is getting up to sharpen her pencil*)) 25 percent, blah blah blah. Which one paid the most?

Those are just strictly crankin out numbers, for the most part. Being able to change decimals to percents, percents to fractions, being able to compare, contrast ‘em. Who’s bigger, who’s smaller. Simply understanding what’s a good estimate for this. Those. There are ten questions and the number, oh excuse me, where it says total earned is the total number that you got correct out of that. In order to be: academic recognition, you must pass each objective so if you have six or less, you cannot get academic recognition, you need to pass each one. The second one is: algebraic reasoning. Those are the ones where there giving you the list and you're writing the expression. What's the twentieth term? Those kinds of questions. There are ten of those on every TAKS test. Again, you need to get, um, I think you have to get eight out of the ten, excuse me, correct for academic recognition. We, if you have problems in those two, you’re probably gonna have problems in other places. Turn around and stop, Brian. (2) Turn around and stop. So, uh, you need to, you need to be able to see that. The next one is geometry and spatial reasoning. That’s the, given this, what’s the, which of the following would be the building? You know what I mean? They give you the top view and then they ask you to build it. Uh, what’s the right side view of this? Also in: that objective, is the Pythagorean theorem, which we have not done, we’ll be- (3)

S: Brian ((*Ms. M waits for Brian to turn around*))

Ms. M: Covering that next week. So if you missed a lot in there, I’m gonna say that at least three of those from Pythagorean theorem, so I would have expected you to miss three of those. Pythagorean theorem also deals with square roots, uh, stuff that we have not done. The uh, objective four is the one that concerns me the most. That’s the measurement one. Included in objective four is surface area, volume, um:, what happens when you double the length, what happens to the area, if you, you know, have a scale factor of three:, what's gonna be the change in the volume, those kinds of problems is measurement. The other big thing about measurement is there’s only five questions (4) Thank you ((*visitor bringing something to T*)) So if you, miss two, of those five, you’ve just failed that objective. (3) Right? Cause if you only get three out of five, that’s only a 60 percent. So there’s only five questions. Every single one of them is really important. So starting today and for the forsee-, for the next week or so, we’re gonna be doing the bottom, at least one of the bottom questions. We’re still trying to do calculations, we’re still not bright shining stars there, but we’re also looking at that bottom TAKS question is directly out of the measurement section.

S: so if you failed, do you automatically have to take the tutorial?

Ms. M: What we did, remember that 60 percent is passing, so if you got 31 correct, They, on that, on that paper, it says 35, that’s for a 70, but we know that 70’s not passing. So if you got less than 31 (1). It depends. If you got 30, I may or may not have invited you to tu-, tu---

S: --So if I got, I got 31. So if I got 31 I do not have to take tutorial?

Ms. M: You do not have to take tutorials, cause we're gonna assume that we haven't done Pythagorean, we haven't done, uh, mean, median, and mode. We haven't done graphs. We haven't done probability. So those are things you haven't even been taught yet plus the week, week and a half before TAKS, we're just gonna do, you know, the format, practicing, highlighting, defining words, stuff like that. So I'm gonna assume if you can get 31, (1) then you're probably good with just being in class and doing your homework and stuff. Right, if you know::, you would know at this time if you're being invited for tutorials ((*warm-up buzzer goes off*)) (3) you would know. Okay. (1) So does that cover that? All that? Or is there still more questions about that?

S: What if we want to come in for a couple of sessions?

Ms. M: If you would like to come in to tutorials, you are more than welcome to do what you need to do. However, (2) is, Brian, Brian, Brian--

Brian: --I just, (need to figure this out, I need to) (inaudible)-- ((*Brian is visibly frustrated and has been turning around to get help from the student behind him*))

Ms. M: Then you need to do it quietly, baby. You just do it quiet-, but you, no, please, try if yourself:, that's the whole scam. Ma'am?

S: What was the (inaudible)?

Ms. M: What is the (1), what?

S: (processes)

Ms. M: Oh, processes. That's more like um, (1) um, they're asking you if you know:: what to do, like what would be a reasonable conclusion? Some of, a lot of those ones are underlying processes. What is the missing information? Uh, stuff like that. ((*to class*)) That tutorial, uh, TAKS thing is for you.

S: Are these supposed to be 50 out of 50 or 50 out of a hundred?

Ms. M: 50 out of a hundred, sweetie. (2) Okay, other questions? (1) Go ahead and put your name on it, oh, you already did that. Alright, here we go, the next thing we need to do. If you were here yesterday, you got an assignment. You gotta finish that, finish that assignment. If it's already finished, I loaded up some (1) gizmos: for you. So go ahead, if you've already done the homework from last night, while I'm explaining to them that first section, go ahead and, uh, work the gizmos. No I'm talking to the people, the seven people who were here yesterday.

Bernice: (inaudible)

Ms. M: Who's, two, oh, would, how many (inaudible) oh so two. ((*counting, out loud, how many students need the notes from the day before*)) two, three, four, if you were here yesterday, right and Pedro, you were here yesterday? So Penelope. No, there you go, here you go. (2) All three of you were here yesterday? One. One, two, three. You were here yesterday. This goes to, no, there you go. ((*finished handing out notes*)) Alright, now, listen up (4) What? Will you go get him, so, thank you very much. (4) Yes. Yes. Alright now. Oh, uhm, Carla's coming to get them. Here we go. Now. This is what we need to do. Have a seat ((*to Jeffrey*)) ((*pulling down the screen and pulling the cart in* *front of the class*)) This is what we want to do. There's people in kind of two places, because some people were here and some people weren't. So this is what we need to do. Thing number one. We wanna give you, I wanna give you the notes from yesterday in addition to the notes from today. So this is what we're gonna do ((*Sheila and Mandy are sharpening their pencils*)) (2) we're going to use the highlighters (5), oh yeah, I'll take that, we'll just put it right here, (3) ((to James)) you are so not gonna do that, (1) right? That just didn't take a lick of sense. (2) you're so not gonna throw them. Okay ((*Sheila hands a pencil to Melody, Ms. M puts her hands over her face and shakes her head*)) (9) At this point what it is, is class time to work on your assignment that you're wasting cause I think I can get this done in about twenty minutes (3)

Mandy: It won't sharpen.

Ms. M: I'm sorry. (1) A lot of times, the first thing you need to do is empty the shavings case out. If it's full, you're right, it won't work. If that's not true, I'm sorry. I have no idea what to tell you. (6)

Mandy: Oh my God, it's so: full.

S: Good job Mandy.

Ms. M: First thing you need to check. Alright, this is the deal. We did dilations. Dilations. We were making something, what?

SS: Smaller or bigger

Ms. M: Smaller or bigger, right? What was true about the shapes when we dilated them?

SS: (inaudible)

Ms. M: They were similar figures. They had the same angles but different (2)

SS: Side lengths

Ms. M: Side lengths. Okay, what we're doing now with similar figures is, we're saying okay, we can kind of find out what the scale factor is we kind of know what dilations means. Now lets see if we can find the missing side if we know they're similar ((*Mandy is just sitting down from sharpening her pencil*)) We know the angles are the same and we know the side lengths are different. Now you kinda did some basic similar figures in seventh grade. So, obviously, now, there going to extend it just a smidgen, it's not gonna, excuse me, it's not gonna be exactly the same. But what I wanted to do was review what we'd already done and then take it to where we wanna go into the eighth grade. So here we go. First thing we wanna do is take our blue highlighter (1) and we're going to find the side:: that has the variable on it. Which side is that?

SS: (inaudible)

Ms. M: Line segment what? Sir?

James: EF

Ms. M: EF has the variable, the variable means the (1) letter, and what I'm gonna do is I'm going to highlight it,

S: Highlight that line?

Ms. M: Highlight that line, cause that is the line that is length x that I need to know. Then in the same color blue highlighter, I'm going to highlight the same line: on the other figure. What line would that be?
SS: BC
Ms. M: BC ((*the students did not know how to respond to the earlier question about identifying the line*)) No problem there. (3) Now there are different ways to set up proportions. I'm gonna do it the very easiest. I'm going to take the, the uh, whatever is on the left-hand side that's blue, which is what?
SS: Thirteen
Ms. M: Thirteen. What ever is blue on the right hand side is?
SS: x
Ms. M: x. I wanna solve this using a proportion. A proportion is two ratios with an equals sign, so I'm gonna give 'em both a fraction bar and an equals sign (5) Then I'm going to pick up (1) my other color marker and I'm going to find two other sides, that is, one from each figure that have numbers with them. Which two sides am I going to color?
SS: ED and BA
Ms. M: (6) ((*highlighting both line segments*)) And then I'm gonna do the same thing numerically for my proportion. On the right hand side I've highlighted this side that has a ten. On the left hand side, I've highlighted the side that has a five (2) So in this case, the left hand proportion ((*ratio*)) is dealing with sides from the left hand figure. The right half si-, excuse me, right half ratio is dealing with sides from the right hand fig-, side. And I can see that across, the sides are the same (2), right? (1) Okay. Now, so I've said on my proportion that I know it's right, I can kinda see that it makes sense to me. Now what I want to do is I want to solve it, so over here in the solving side, I'm going to look at that and try to decide, let's see, I'm just trying to find x. ((*allowing students time to work on solving the proportion for x*)) (18) And what is x?
SS: (several inaudible answers) (26)
Ms. M: 26. How'd you do it?
S: (inaudible)
Ms. M: Five times two? We're working like there are two fractions, right? Two equivalent fractions, so we're working across. So whatever I do to the denominator of one, I have to do to the numerator. Thirteen times two is 26. Yeah. I'll buy that. Anybody do it differently?
Jeffrey: ((*and others*)) I did it the other way.
Ms. M: What's the other way?
SS: Dividing and (inaudible)
Ms. M: Cross- mult-, so solving it like a proportion. So you have thirteen over five and x over ten. Which two numbers did you multiply?
SS: Ten and thirteen
Ms. M: Thirteen and ten give you 130 and--
S: --(inaudible)

Ms. M: Uh huh, and then you're dividing by five, so then you have 130 divided by five, five into one, no, thirteen goes two::, 30, five into 30 goes:--
SS: (inaudible)
Ms. M: Six. Is there a difference on which way you're gonna do it?
SS: No/Yes
S: They both get the same answer.
Ms. M: They both get the same answer. If you’re calculations are correct, they both get exactly the same answer. Does it matter which one you do?
SS: No
Ms. M: No. We know:: that sometimes we can go across. (1) We know that always: cross-multiplication will work. So when in doubt, you always need to know how to cross multiply, but if you see it across first, that is fine. (2) Questions? (3) I need a volunteer. Can anybody do, um, the second one? Go ahead Sheila. Go ahead and do the second one for us so we can see what we’re doing. ((*to Linda while Sheila writes her solution on the overhead transparency*)) See you only wanna do just one side, so just do this side and like this side. Cause these ones, you really don’t know what they are and you're not using those numbers. Do you know what I mean? Like here you're using the 32 and the eight but you're not using this, so all I should see are those two sides. ((*to class*)) Yes, you can do it on paper. Make sur-, oh you're talking about, (2) yes. Oh, and actually hold on, because you might not be able to do this. What I did was, I have little score sheets for you.
Bernice: So we don’t need our paper?
Ms. M: Well, you’ll need to put your answers on here, you might need to do calculations on that. Are you doing the gizmos? Did you already do the gizmos?
Randall: This is the gizmo, right?
Ms. M: No that is not the gizmo. The gizmo is on explorer learning.
Randall: Oh, cause that’s (inaudible)
Ms. M: No. The gizmo is on explorer learning. (8) ((*walking around handing out the score sheets*)) Um, let’s look ((*at Sheila’s explanation*)) Okay, let’s look. Let me do the, um, oh, here we go right here. So we have (2) the x and the two, so the x and the two ((*highlighting the sides*)) (4) and we have (4) the 32 (2) and the eight. (1) The two that um, (1) wha-, what’s the question again? (5) What’s the question?
S: Oh, I said (inaudible) could go over eight or does it really matter?
Ms. M: No, it doesn’t, it does not matter, it does not matter as long as the blue are across from each other or their up and down from each other, it doesn’t matter. Like some people like to do it, the blue:, this is green. (1) The blue on one side (4) and the green on the other ((*Showing different ways to write the proportion*)). And that’s fine:. It doesn’t make a difference. (1) you either have to have the same side across from each other. Or you have to have it up and down and it doesn’t matter which one’s in the numerator and which one’s

in the denominator. As long as across, (3) they match. Do you know what I'm saying?

S: Yeah. So it like, cant' be, like, uh, two over x, right?

Ms. M: It could be two over x. That would be fine, but what would have to be true of the green one?

S: It would have to be eight--

Ms. M: --uh huh--

S: --over 32.

Ms. M: Right. You either have to have, like two and eight are the same shape, so two and eight. You have to have either the same shape across or, uh, yeah, right. You have to have them either horizontal or vertical. That's the key to it. (3) Okay, questions? (1) Can anybody do the third one? (2) Where are we at? Yeah, the third one, can we, go ahead Amber. Gosh Ka-, um, I'm sorry. You just keep sayin it, you keep doin it, but I keep not seeing you. it's like man::, what did I do to make her mad? She won't let me work.

Bernice: (inaudible) ((*about logging on*))

Ms. M: It should be first initial, last name and whatever password that you chose. I tried to make you guys do, for your logins, first initial last name and whatever:, um, uh, password that you chose. That was my intention (8).

Jeffrey: She messed up the, uh, she messed up.

Ms. M: She messed up? How'd she mess up on it?

Jeffrey: (inaudible) on that, she put the x and the five(inaudible)

Andrea: I didn't take (inaudible)

Ms. M: Right. See that's the thing. We, we kind of gave some of our pre-APs in seventh grade the twisted ones, but for the most part, we left 'em like one and two. So now we're gonna get seriously into the twisted, would not be unusual.

S: Can I do the next one?

Ms. M: Sure.

S: (inaudible)

Ms. M: that's not it? Let me see, Randall, if I can pull you up and, the-, are you guys having, you know the ((*Randall is having problems getting logged on to the system. Ms. M has a list of usernames and passwords*)) Oh, when I registered you, I tried to tell you to be first initial last name. (1) oh, you know what? Mm, mm, mm:. I have a loaner ((*computer*)), so let me think where, what is it, Explorer learning?

Randall: yes, ma'am.

Ms. M: Is it www.explorerlearning?

Randall: yes, ma'am.

Ms. M: and the, is anybody else's computer extremely slow? Brian, you are not there. Close it. Put it away.

Brian: yeah I am.

Ms. M: yeah, but you're not supposed to be. You were absent yesterday. The only people, were you here yesterday?

Brian: No. I just--

Ms. M: --Well stay with us, then. They were here yesterday. They already did it.
S: (inaudible)
Ms. M: Okay. Go ahead, Katie.
((*Brian and Mandy talking about the assignment*))
Ms. M: yeah, wait on it, okay? I have the, the next, the other thing, for those of you who were here yesterday, as soon as we do this. I'm gonna go into today's notes. So you're gonna need to come back with us. Okay? (1) Just in case, (5) explorer learning dot com, right?
Trisha: I put my first name (inaudible)
Ms. M: Okay, hold on, hold on, hold on. one thing at a time. One thing at a time, one thing at a time. And this machine is so:: slow. Is it just slow, or is it just my machine?
S: Mine is slow, too.
S: Mine's fast.
Randall: It tends to be that Safari's faster than explorer learning ((*Internet Explorer*))
Ms. M: I'm always in explorer learning. I'm an explorer learning girl. I like it. Questions, problems, things we don't understand on these? ((*back in front of the class now*))
SS: (mumbling, with some audible 'nos')
Ms. M: Do we want to be highlighting? Yes. I do like it when you highlight 'em, just because as we begin twisting them and doing all kind of stuff, it's kinda nice for you to visually see: (1) that coloring. That's just kinda nice. So we can get sides to go together, okay? I do not expect these problems to give you a lot of trouble (3) Okay, I don't. I just don't. So let's go to the good stuff. (1) Yeah, the hard stuff. We're tired of this baby stuff. (1) If you were here yesterday, you are comin back with us, because we're now going into the new stuff so:. Yeah, I'm still waiting to try to pull up your um, passwords and stuff. One, two, three. One, two, three. ((*counting out under her breath as she is passing out notes for the topic*)) One, two, three. Got 'er all squared away?
S: I'm getting the wrong answer.
S: I'm fine.
Ms. M: You just not setting up the proportion right or what's the scam?
S: Yeah
Ms. M: That's it. We gotta color code it. Alright. Oh:, dog gone it. Ahs-, okay. (1) Um:, (5) Alright, so here we go. (1) This time, we're talking about what I ca-, I don't know. They have-, probably have an official name. I call these nested figures.
S: Nested
Ms. M: Nested, because one is
S: Nested inside the other one
Ms. M: Nested inside the other one. Now, here we go (3) ((*drawing a nested figure on the overhead*)) There are two ways to do these. Two ways to do these. Okay? We're gonna kinda look at both ways. The first thing is, since we just did this, just did this, right? Let us see if we can do it nested. If we can't,

	we'll take them apart. Pedro, are you with us? You need to um finish that and then. So here we go. Yep?
Jeffrey:	How is that possible (inaudible) ((*referring to the drawn figure*))
Ms. M:	You know what? I absolutely believe that highlighting it will clarify that in your mind.
James:	I was about to say, would it mean that there's ten below the lines and above the lines?
Ms. M:	Yes. So here we go. Are you ready?
S:	Yes
Ms. M:	Is everybody ready?
SS:	Yes/uh huh.
Ms. M:	Let's do the blue highlighter is the variable and it's corresponding side. Which two am I going to do?
SS:	(the x and the six)
Ms. M:	Yes, the x. Who, alright here we go. The x and the six. DE and AC. Yes?
SS:	Yes.
Ms. M:	Now here::, when we do it like this, we might, you know we were doing the two different figures, we might just do: the six over the x, right? We might just do blue together.
S:	Yeah
Ms. M:	I mean, we might. It doesn't really matter as long as we do it all the same, one way or the other. So I have the small one over the big one. (1) Now I need green, right? What's the green for the small triangle?
SS:	(ten)
Ms. M:	Well, what letters? What line segments?
SS:	EB and EC
Ms. M:	BE, from here to here, yes? ((*highlighting line segment BE on the overhead*))
SS:	Yes
Ms. M:	BE
S:	Oh, so we're not doing (inaudible)?
Ms. M:	Well, I'm doing the small triangle and then the big triangle. This is the, this is this one for the small triangle. Can you see that? (3) Okay.
S:	(inaudible)--
Ms. M:	--then which one is for the big triangle? Yes ma'am?
S:	BC
Ms. M:	BC So all the way from B ((*highlighting BC*)) down to C.
S:	Now would you add the segments together?
Ms. M:	So that is (1) twenty. (2) So the number for the small triangle is?
S:	Ten
Ms. M:	Ten, and the number for the big triangle is?
SS:	Twenty
Ms. M:	Twenty
S:	Oh, you just add them together?

Ms. M: Right, because what it's saying is from here, what it's saying is that these two tens, it's saying from here to here, from B to E is ten. So if I walk from here to here, I've walked ten, but when I get to E and I go to C, how much farther do I have to walk?
S: Ten more.
Ms. M: So if I go from B and I get all the way down to C, how far have I walked?
SS: twenty
Ms. M: twenty. That's where it comes from. Okay? (2) Get it out of your mouth ((*gum*)). (4) How we doin? Does that help? See when you go to highlight it, I think it helps, just because you can see it in your head. Does that help?
SS: yeah
Ms. M: okay. Do we want to look at the next one? Can you solve the proportion, or do we need to solve the proportion?
S: (we can solve 'em both)
Ms. M: Let's make sure we're right. Okay. Six to ten. I, I, I can't multiply across, so I'm gonna have to bring it down. Six times, what am I multiplying six by?
SS: Twenty.
Ms. M: oh, six by twenty. That's gonna give me 120, right? And then I have ten by?
SS: x
Ms. M: x. so I'll need to divide both sides by?
SS: Ten
Ms. M: Divide both sides by ten. Let's see. 120 divided by ten is?
SS: Twelve
Ms. M: (2) Woops, hello. (3) Twelve. So x is equal to twelve units: right? Twelve whatever it is. The distance. (2) No? Yes? Maybe? Okay Miss?
S: I always start with the little one.
Ms. M: You always start with the little one, the easy ones, Miss. Whoever is, my uh, uh, my D-hall people are just doing a lousy job on cleaning my transparencies. I have to tell ya. (2) I'm gonna have, I think I'm gonna have to start inspecting 'em. Not letting 'em (1) get away with such shoddy workmanship. Shoddy workmanship.
S: Shoddy workmanship.
Ms. M: Shoddy workmanship. Alright. Here we go. (2) We are going to first highlight and what color are we highlighting for our um?
SS: Blue
Ms. M: Blue for the x, right? That's cause we always, we know:: that's what we're doin. Solving for the x. we might as well start with what we're looking for, what we are looking for. Hello. And who goes with x?
S: FE?
Ms. M: FE or EF? So we're gonna highlight that in blue? ((*highlighting EF*)) And this time do you want to do it across? Or do you want to do it up and down?
SS: Across:

Ms. M: You wanna do it across? So the x is the big one. I have to remember that. I'm just gonna put big and then the two is the small one. Okay. ((*setting up the proportion on the overhead*))
S: I thought it would be x over two.
Ms. M: Either way. Either way. Either way. We'll do this one x over two so you can check, oops, hello, so you can check yours. Okay? It doesn't matter which way you do it. You can either do the same side across. Or you can do it up and down. The question is, which other two sides am I highlighting?
S: Uh, FG
Ms. M: FG: and CD. Now, let me think. Where does the te-, if I'm going, this one over here, this left hand one, x, x is over who?
S: x is over, uh, 40.
Ms. M: 40, because the x is from the big rectangle, so the 40 has to be from the big rectangle. And thank goodness, I put a little b to let me know the big: one goes up and down. So the two goes with who?
S: Ten.
S: The two goes with the::
S: Ten:
Ms. M: Ten. (3) Okay. Now, the question is on this one where you do it up and down, whose gonna be in the numerator?
SS: Uh::,
Ms. M: Forty or ten?
SS: Ten.
Ms. M: The x comes from which one?
S: (inaudible)
Ms. M: The x comes from the::?
S: Big
Ms. M: Big rectangle, so what has to go in the numerator over here?
SS: Forty
Ms. M: the forty, the big: rectangle. It doesn't matter which way you do it, but you gotta do it correctly. Either the big rectangle is up and down, or, in this case, it's across. So the two would go across from the ten. Now, sometimes: what happens is people start looking at these and they like it better when the x is on the other side, (1) It is true that these are equal, it doesn't matter which one's on either side, because I might be able to see from here, but I can definitely, if I put 40 over ten on the left and x over two, (2) then I can look from ten to two and say that I'm going to do what? Woops, hello.
S: Divide by five
Ms. M: Divide by five, and if I divide by five across here, what am I gonna do?
SS: Divide by five
Ms. M: So x should equal?
SS: Eight
Ms. M: Eight. Most of us have a hard time doing it when the x isn't on the right hand side, so is it true that I can put the two over, as long as I leave them the same,

like that fraction stays the same. The two over ten, and just move the x and 40 over here:. Can I do that?

SS: Yes.

Ms. M: Yeah, and when I do, it's much easier for me to see. What do I do to get from ten to 40?

SS: (inaudible)

Ms. M: Multiply by:?

S: Four

Ms. M: So what am I gonna do to this one?

SS: Multiply by four.

Ms. M: Same thing. (2) Okay? Now, so, uh, so when you're doing this, not only can you write it in different ways, but if you want this one on the right ((*the fraction with the variable*)) because it's easier for you to see, that is fine as long as you keep the same numerator, you can't put the, switch the numbers around. (3) Alright. I think we have.

S: It's not really that hard

Ms. M: Say it again.

S: (inaudible) It's actually kinda fun (inaudible)--

Ms. M: It's kinda easy? So lets just, and this is what we wanna do. Three on the bottom and four on the top on the other side. Let's just setup the proportion, okay? Let's just set up the proportion. Can we do that? And then we'll do the angle ones real quick and then I'll give you your homework assignments. How's that? (4) Let's just set up the proportions real quick and then I'll make 'em so someone can put their answers up their and see what we got. (5) Let me turn the lights on so yall can see what your doing and I can see what I'm doin cause I can't see what I'm doin. (3) Well everything's easy if you know how to do it.

S: Or if you're learning how to do it.

Ms. M: That's true ((*Ms. M is drawing the figures on transparencies while SS are working on setting up the proportions for the two problems*)) (29) Ma'am? ((*Mercedes is whispering her questions*)) Marisa, are you getting any of this or where are you with us, honey?

Mercedes: (I'm getting it)

Ms. M: Are you?

Mercedes: Mm hmm.

Ms. M: Good. Excellent. (1) Okay, I need a volunteer for the third one. ((several students call out to do the problem at the overhead)) Go ahead, Tony. (3) I know, I'm getting my act together. It's slow, but it's steady. There you go. See. See my act is almost together.((*laughs to herself as she walks off*)) (4) Okay, if you::, if (2) you have the two shapes together, go ahead and see if you can do the ones with the angles which are the ones that might: confuse you. Might: confuse you (21)

S: So we're gonna try the angles out?

Ms. M: Yep. If you got, if you have both those other ones and you're just sitting there being bored? Yes, absolutely. (3) What do you know about the angles?
Bernice: That a triangle equals (inaudible)
Ms. M: What do we, angles of:: similar figures?
S: 90 degrees
S: They have the same angles.
Ms. M: (1) Right. Cause if we change, if they're the same shape, can we have different angle measures?
SS: (inaudible)
Ms. M: Uh huh.
Randall: The sum of the angles: will--
Ms. M: The sum of the angles of a:?
Randall: (inaudible) 180 degrees?
Ms. M: Triangle
Bernice: 180. I added those up and
Ms. M: But you know what? (1) eight minus two isn't four
Bernice: Oh:
Ms. M: It was gonna be, but then they changed it. Okay, so here we go. Let's look. Is this right?
SS: yes
Ms. M: Nine over three. Nine and x. So the big one's across in this case. Cross multiply, divide by three, six, sure. I'm buyin it. Definitely true story. How bout? Do we have this? Oh, do you guys still need this?
S: No
Ms. M: Any that can do this one? ((several students frantically raising their hands)) Go ahead, Carla. Go Carla, go Carla ((*singing and doing the Cabbage Patch*)) (6)
S: Can you please (write the angles for these)?
Ms. M: Say it again.
S: (inaudible)
Ms. M: Mm hmm. We're just trying to make sure you understand that uh:
Bernice: What do we highlight?
Ms. M: You wouldn't have to highlight anything (2) Who was here? One, two. You already have this. Two people were absent. I'm trying to--
Bernice: Marisa was here, too (inaudible)
Ms. M: One, two, three, four. Everybody in this row was gone.
S: Is this for homework?
Ms. M: Yes. One, two. Three. You wanna make sure that your other answers are correct however. (1) One. Uh, yes.
S: (inaudible) gonna take the test?
Ms. M: Yes
Jeffrey: And can I get a (inaudible)?
Ms. M: Can you get, uh, sh-, graph paper, uh, gra-, yes.
S: Are we gonna learn (how to do the) angles?

Ms. M: Yes. We're going to. As soon as he's done. We're gonna have about two minutes. We'll be able to do it.
S: (inaudible)?
Ms. M: Oh, don't have a clue. Sorry. (8) Oh but wait, wait, wait, wait, wait.
Carla: Did I not put the numbers right?
Ms. M: You did not put the numbers right, cause look. x and eight. Here's x and here's eight. Yes?
Carla: Uh huh.
Ms. M: x is from the which one?
Carla: Uh:, this one. The small one.
Ms. M: So--
Carla: --five should be with this.
Linda: Wouldn't it?
Ms. M: Wouldn't it what?
Linda: This one and this one confused me, but I get the other ones. Like, why wouldn't you do a--
Bernice: --(draw a triangle in)--
Ms. M: ((*to Bernice*)) Wait, wait, wait. Let her speak. Let her speak. ((*to Linda*)) Why wouldn't you do?
Linda: Why wouldn't you put –
Ms. M: --Go ahead. Show me.
Linda: Eight, x, and like I don't know what to do with the other ones.
Ms. M: Okay, this is the deal that you have, this is the deal that you have. The eight is with the what?
Linda: The big one?
Ms. M: Mm hmm. So I need this whole side from here to here
Linda: Ten
Ms. M: So it's ten.
Linda: So::
Ms. M: And with the x:
Linda: The colors are messing me up on that one. So, on the other ones you just put the two like (colors)--
Ms. M: --Right and that's why I put this, it's all the way. It's this whole way.
Linda: Okay.
Ms. M: Now which side goes with the x?
Linda: (inaudible)
Ms. M: Yeah, cause it's just from here to here
Linda: So then like, when they're together like that with one side, then (inaudible)?
Ms. M: If it's that whole side, right. Cause this whole side goes with it, this whole side goes with it, and this whole side goes with it, too. So you would have to, like, if you did it, you would have to do all the way from A to C.
Linda: Alright.
Ms. M: Does that make sense?
Linda: ((*nods*)) Yeah.

Ms. M: Okay. Alright, here we go. So x should be four, yes? Listen up. Let's very quickly look at the angles. I think that the problem with the angles is you guys are trying, now you want it to be more complicated. Okay, well. Huh. We know: that the angles in similar figures do what?
S: um, stay the same.
Ms. M: They stay: the: ?
SS: Same.
Ms. M: So if this one's thirty, what's this one?
SS: Thirty.
Ms. M: Oh, okay. What's this one?
SS: Ninety/that's ninety
Ms. M: What do I know about the angles of a triangle?
SS: (inaudible)
Ms. M: Right angle is ninety. What else do I know?
S: All the sides have to add up to (inaudible)--
Ms. M: --All the angles equal:?
SS: 180
Ms. M: All the angles on the inside equal 180. So I have 90 and I have 30. How much is that?
SS: 120
Ms. M: So which, what does this ((*the third angle*)) have to be?
SS: 60.
Ms. M: and if this ones 60, what do I know about this one?
SS: It's 60
Jeffrey: So all of them have to be 180.
S: Practically, all you do is (inaudible)--
Ms. M: ((*to Jeffrey*)) The a-, the inside angles of a triangle always add to 180
Jeffrey: Tri-, here, I'm writing it down, Triangles--
Ms. M: Sum of the angles of a triangle always equal 180. ((*student comes in and hands Ms. M a note*)) Thank you, sir.
Jeffrey: Then on the squares:, (inaudible)--
Ms. M: --Well:, the problem with this, this one, if this one, oh, what's this one? ((*pointing to the angle on the parallelogram, opposite from a given angle*))
SS: 120
Ms. M: 120. and what's A?
SS: 60.
Ms. M: (2) Right? Now, if this is, oh, what's F?
S: 120
S: F is six-, 120 cause B is 120--
Ms. M: You know why, cause this one and this one have to be the same. This one and this one has to be the same. This one and this one has to be the same. What, these two are the same. What do you suppose is gonna be true about these two?
SS: They're the same.

Ms. M: So what's this one gonna be?
SS: 60
Ms. M: 60::?
SS: Degrees.
Ms. M: Thank you. And this one's going to be:?
SS: 60 degrees
Ms. M: Okay. That's the really, really, really hard part. Right, we're, right, right, we're just finding out what degree measures they are. That's all we're talking about.
Jeffrey: How would you figure it out if we just had the 90 degree angle?
Ms. M: You wouldn't. You wouldn't be able to tell. They would have, for a triangle, they would have to give you one other angle.
James: So would it be the same if they had the one 90 degrees (inaudible)
((*bell rings*))
Ms. M: Oh they could be anything else as long as they ad up to another 90. Do you know what I mean? You have, cause you have a 90 degree angle, so you have another 90. It could be 1 and 89:, 2 and 88. just (inaudible). Half and 89 and a half. Alright. People.
Mandy: I need the homework.
Ms. M: Okay wait. I, uh, she had it, she just passed it back to you.
Bernice: I did?
Ms. M: (inaudible) the homework. I passed it back to you. Now you have two. Okay, listen up. You should have, Hey:: People, people. You should have your homework assignment. Both sides of that. Due tomorrow when you come in. People, I need my markers back. People please::, yes I wanna see the pictures. Have a good day. I'll see you tomorrow.
47:50

APPENDIX C

WEBQUEST WEBPAGE

A Screaming Good Time!

WebQuest: A Middle School Graphing Adventure

  

Introduction - Task - Resources - Process - Evaluation - Conclusion - Home

Introduction

When is the last time you visited a theme park? What was your favorite ride? The bumper cars? Waterslides? Roller coasters? One of the most popular rides in a theme park is the roller coaster. People are facsinated by the height, length, and speed at which roller coasters travel. There are people who travel the world to experience the thrill of a new roller coaster. The internet is even filled with websites dedicated to them. Come along for the thrill!

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Task

Your class has just won an all expense paid trip to one of six theme parks from ***Roller Coaster Magazine***. The essay your class wrote about their love of roller coasters really impressed the judges. The problem is how to decide on the ONE theme park your class will choose.

Your task is to research and record data on six roller coasters and decide which roller coaster offers the best thrill based on height, length, and maximum speed. Based on the results of your research, you will try to convince your class to choose the theme park you feel they should visit. In order to convince them, you will create a line graph to help better state your case.

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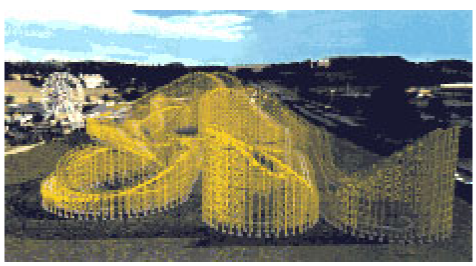
Resources



Dragster: http://www.topthrilldragster.com/public/inside_park/rides/thrill/ttd/specs/index.cfm

Montu: http://www.ultimaterollercoaster.com/coasters/yellowpages/coasters/montu_bgt.shtml

Superman: the Escape:
http://www.ultimaterollercoaster.com/thrillrides/pictures/superman/index.shtml

X: http://www.sixflags.com/parks/magicmountain/rides/x.asp?id=rides&rideid=thrills

Deja Vu: http://www.sixflags.com/parks/greatamerica/rides/dejavu.asp?id=rides&rideid=thrills

Mr. Freeze: http://www.sixflags.com/parks/stlouis/rides/freeze.asp?id=rides&rideid=thrills

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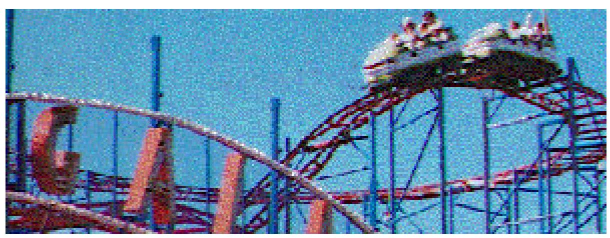
Process



The class can not decide which theme park they should visit, all expenses paid, as the result of winning Roller Coaster Magazine's Middle School Roller Coaster Contest. Students will break up into pairs to research six roller coasters at nearby theme parks to see which roller coaster would give the best ride based on height, length, and maximum speed. Based on this information they will vote on which park to visit.

1. Choose a partner for this investigation.

2. Your teacher will give you a data collection sheet. If not, go to Data Collection Sheet and print it. Use this worksheet to collect your data.

3. Use the internet sites provided to locate actual heights of your roller coasters.

4. Calculate the mean, median, mode, and range for the actual height of each roller coaster.

5. Use the internet sites provided to locate the actual length of your roller coasters.

6. Calculate the mean, median, mode, and range for the actual length of each roller coaster.

7. Use the internet sites provided to locate actual speed of your roller coasters.

8. Calculate the mean, median, mode, and range for the actual speed of each roller coaster.

9. Create 3 line graphs to show actual heights, lengths, and speeds. (Graphs need title,
name axes, and a key). Go to How to Make a Line Graph in Excel.

10. Choose a theme park based on the roller coaster data you collected. Write a short paragraph that describes why you chose the theme park based on your research.

11. You will turn in your paragraph, line graphs, and data sheet after the Conclusion is complete.

12. Move on to Conclusion.

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Evaluation

Your "A Screaming Good Time" WebQuest will be worth 32 points.

You will be graded on these four criteria:

1. You will be assessed on your ability to use computer time wisely.

2. You will be assessed on your ability to organize data in a table.

3. You will be assessed on your ability to calculate the mean, median, mode, and range for height, length, maximum speed.

4. You will be assessed on your accuracy in making your line graphs of the statistics of your roller coasters

Go to "A Screaming Good Time" Scoring Rubric and print to use as your guide.

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Conclusion

After visiting 6 different amusement parks and researching six different roller coasters, have you made your decision? Were speed, length, height enough information for a class of roller coaster enthusists to make a desision? What other information would you need? If you feel like you need more information, download the Challenge Data Sheet and research additional information that would help your class make a better informed decision about the roller coaster they would choose to visit compliments of *Roller Coaster Magazine*.

Share the results of your data collection, using your graphs, to your class. If you chose to do the challenge exercise, please share that information as well. Explain to the class which roller coaster you chose and why your class should choose that park.

After all of the groups present, take a class vote for the roller coaster they will select as the result of winning Roller Coaster Magazine's Class Roller Coaster Contest.

Congratulations and enjoy your trip!

*Teachers: Have students graph the results of the class vote for their favorite roller coaster. If more than one class is doing this activity, display the results for each class. Compare the results of each class.

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EDUCATION

***Ph.D.*, Texas A&M University**, 2005 – College Station, Texas
Ph.D. in Curriculum and Instruction.
Dissertation "Authoritative Discourse in the Middle School Mathematics Classroom: A Case Study"
Co-Chairs Dr. Gerald Kulm
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***M.Ed.*, Texas A&M University**, 2004 - College Station, Texas
M.Ed. in Curriculum and Instruction.

***M.A.*, East Carolina University**, 1999 – Greenville, North Carolina
M.A. in Mathematics.
Thesis "Applications of Jordan's Theorem to Operator Theory"
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B.S. in Applied Mathematics.

EXPERIENCE

2003-2004	Mathematics Education Instructor, **Texas A&M University**, College Station, Texas
2002-2003	Teacher, **Huntsville High School**, Huntsville, TX
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